

Persistent Blessings of Luck*

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First Draft: Nov 11, 2016

This Draft: June 8, 2018

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Abstract

Persistent fund performance in venture capital is often interpreted as evidence of differential abilities among managers. We present a dynamic model of venture investment with endogenous fund heterogeneity and deal flow that produces performance persistence without innate skill difference. Investors work with multiple funds and use tiered contracts to manage moral hazard dynamically. Recently successful funds receive continuation contracts that encourage greater innovation, and subsequently finance innovative entrepreneurs through assortative matching. Initial luck thus exerts an enduring impact on performance by altering managers' future investment opportunities. The model generates implications broadly consistent with empirical findings, such as short-term performance persistence and long-term mean reversion, outperforming funds' appeal to innovative entrepreneurs even with worse terms, and the link between failure-tolerance and innovation. Initial luck may also amplify the effect of innate skill differences.

JEL Classification: G10; L26; O31; G24

Keywords: Endogenous Heterogeneity, Deal Flows, Private Equity, Venture Capital, Innovation, Delegated Investment, Manager Skills, Dynamic Contracts

*The authors thank Ulf Axelson (discussant), Douglas Diamond, Vincent Glode, Will Gornall, Zhiguo He, Yael Hochberg, Ron Kaniel, Steve Kaplan, Leonid Kogan, Josh Lerner, Ramana Nanda, Jimmy Oh (discussant), Dmitry Orlov, Alberto Plazzi (discussant), Matt Rhodes-Kropf (discussant), Philip Valta (discussant), Lucy White (discussant), Rob Vishny, Mindy Zhang (discussant), Jeff Zwiebel, as well as the participants at NBER Corporate Finance, Adam Smith Workshop, European Finance Association Meeting, Luxembourg Asset Management Summit, Northeastern Finance Conference, Frontiers of Finance Conference, Private Market Research Conference, PCRI Workshop, ABFER Annual Conference, CICF, AsianFA, EFMA, and seminars at Chicago Booth, Chinese Academy of Sciences, CUHK Business School, City University of Hong Kong, Rochester, CUHK Shenzhen, and SAIF for helpful comments.

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1 Introduction

Financial economists have long debated whether or not investment managers differ in skills. Many studies of individual stocks, mutual funds, and other fund classes generally find that investors do not consistently outperform passive benchmarks after-fee and out-performance is not persistent (e.g., Wermers (2011)). An important exception is the private equity (PE) industry, most notably venture capital (VC) funds. Kaplan and Schoar (2005) show in their seminal study that VC firms typically manage sequences of funds, and the performance of one fund predicts the performance of the subsequent fund. Harris, Jenkinson, Kaplan, and Stucke (2014) confirm the phenomenon with more recent data. Korteweg and Sorensen (2017) find long-term persistence in expected net-of-fee return spread. Beyond the fund level, performance persistence also exists at the investment level (Nanda, Samila, and Sorenson (2017)) and the individual partner level (Ewens and Rhodes-Kropf (2015)). A widely-adopted interpretation of such performance persistence is that VC managers differ in their abilities, with the more skilled managers consistently outperforming the others.

We link investors, fund managers, and entrepreneurs in a dynamic theory of venture investment to challenge and complement this conventional wisdom. Our key argument is that a temporary shock may have an enduring impact on future investment opportunities. Endogenous deal flow is a salient feature for private equity, especially venture capital: Funds offer capital to promising deals, and entrepreneurs decide which offer to take (Kaplan and Schoar, 2005; Hsu, 2004; Sorensen, 2007). We model this in a general equilibrium model, emphasizing the complementarity between luck-induced fund heterogeneity and future endogenous deal flows.

To illustrate, suppose a fund manager can exert effort to improve a project's probability of success, and his investor finds it optimal to incentivize his effort by rewarding him with better contracts in future. If he is lucky in the current fund and successfully nurtures a project, he then finds it easier to raise the next fund with investors less sensitive to short-term performance. He is consequently more tolerant towards initial failure and experimentation, making his fund more attractive to high-quality innovative projects. This positive reinforcement can lead to persistence in differential performance, both before- and after-fee, across

managers even when they do not differ in skills.¹

Specifically, our model features a group of entrepreneurs born with projects in each period who seek financing and value-added services from fund managers. Conventional projects succeed with higher probability but have lower payoff, relative to innovative projects which are in limited supply. Fund managers, homogenous in our baseline setup, are matched with entrepreneurs who prefer those who are more tolerant towards initial failures and experimentation.² Managers then decide on whether or not to exert effort to improve the chance of a project's success, before the project pays off. Finally, investors invest in funds, allocating contracts based on the managers' track record, using terms contingent on current fund's success or failure, as well as continuation contracts for future funds, to motivate managerial effort.³

In equilibrium, funds have endogenous heterogeneity in the sense that their contract terms are endogenous and distinct. That heterogeneity in turn leads to endogenous investment opportunity sets because projects are assortatively matched to funds. In particular, investors offer a hierarchy of contracts which differ in agency rents. The ones with higher agency rents are more tolerant towards failures and experimentation, and choose innovative projects with higher expected payoffs. Others are matched with conventional projects. By using promotion from lower- to higher-tier contracts, demotions from higher-tier to lower-tier contracts, or terminations of contracts, investors can incentivize managers to improve project success more cheaply. Each individual contract is shaped by the aggregate market environment such as the supply of truly innovative projects, as well as the performance of other funds.

Our model predicts that VC firms with earlier successes are more likely to raise capital

¹In spirit, this paper is akin to Berk and Green (2004): they argue that the lack of persistence in returns does not necessarily mean differential ability across managers is non-existent or unrewarded; we argue that performance persistence and entrepreneur funding choices do not necessarily imply heterogeneity in managerial skills.

²Evidence that VC funds differ in their tolerance for failure and nurturing technology abounds. See, for example, Tian and Wang (2014). Many best performing funds have as high loss rates as average funds, if not higher, and take more innovative nurturing approach. A partner at Andreessen Horowitz, Alexander Rampell, aptly puts it, "You only score home runs if you swing HARD at pitches." Jo Tango from Kepha Partners blogged, "VC is not about minimizing losses" but about taking the risk to create real business. Fred Wilson, co-founder of Union Square Ventures Which has invested in companies such as Twitter and Kickstarter, expressed a similar opinion.

³We assume weaker competition among fund investors relative to manager competition, because this has to hold for any model to generate the observed persistent net-of-fee returns.

for subsequent funds that encourage innovative nurturing and greater risk-tolerance, or more broadly, easier and more frequent access to capital (Gompers and Lerner (1999); Kaplan and Schoar (2005); Tian and Wang (2014)). Expecting greater innovation under top-performing funds, entrepreneurs are willing to accept their funding offers with less favorable contract terms (Hsu (2004)). Moreover, to the extent that project quality is private information for the fund managers or the entrepreneurs, matching with a successful fund signals project quality to other investors through selection, and top-performing funds have an endogenous certification effect even though managers do not have differential skills.

We establish that funding contracts and deals endogenously flow to fund managers who have better track records due to initial luck. Investors implicitly commit future funding contracts to motivate managers' effort, and improve contract terms so as to be more tolerant towards experimentation and innovation. The complementarity between contract term and deal flow implies that under investors' equilibrium contracts, fund performance and investor returns are persistent and predictable, and entrepreneurs willingly accept offers from VC funds with more tolerant contracts. Unlike performance persistence caused by skill heterogeneity, better-performing managers do not increase fees to make after-fee returns unpredictable due to competition from other managers (none of them having superior skills compared with others).

Fund heterogeneity that endogenously arises in equilibrium due to luck can arise in other forms, such as in proprietary network formation, visibility or fund size. Focusing on contracting is a convenient modeling choice that not only articulates the insight, but also allows us to discuss the managerial compensation and thus net-fee performance. We bring together in a simple general equilibrium model three key players in private equity and venture capital: the ultimate investors such as endowments or pension funds, the fund managers, and the entrepreneurs. Consequently, we can characterize the endogenous allocation of deal flows and the evolution of relative performance among funds.

Our model does not contradict the presence of differential manager skill in VC and PE funds. In fact, we demonstrate in an extension that endogenous deal flow and contracting augments exogenous skill heterogeneity under imperfect learning, and thus contributes to

performance persistence. Rather, our baseline setup considers homogeneous fund managers to underscore the point that sheer transient luck can induce the apparent fund heterogeneity, in contrast to innate-skill-based explanations. While alternative channels, such as managerial scale or the ability to commit personal capital after recent successes, can also predict gross-performance persistence, they do not explain net-of-fee performance persistence. Our model offers many testable predictions, and further empirical studies to carefully distinguish between luck and innate skills are called for.

Our findings not only show that it is *possible* to generate performance persistence with little or no innate skill heterogeneity, but also make unique predictions that are consistent with recent empirical studies, suggesting that our mechanism is a *likely* and *important* channel. Our model implies that funds affected by a common temporal shock perform similarly in the future. Our model also predicts that over the long horizon, investment performance is mean-reverting: in the long run, the enduring impact of initial luck will be gradually offset by i.i.d. shocks in following periods. The model also predicts that performance persistence is mostly driven by deal flow, even when there is no skill difference or only *perceived* skill difference among managers. Nanda, Samila, and Sorenson (2017) use the average performance of other funds that share common past productivity shock (year, location, industry etc.) as an instrument to predict the performance of VC firms in question, demonstrating that the VC firms do not have better ability to select future winners or more aptitude in nurturing them to success. Nanda, Samila, and Sorenson (2017) also document intermediate-term performance persistence but long-term mean-reversion that extant theories cannot fully explain. The authors provide evidence that the deal flow and the “access channel” (a form of endogenous manager heterogeneity) explain the majority of persistence, corroborating the mechanism in our model.

Our economic insights should apply more broadly to situations in which a lucky outcome gives a manager an advantage that is self-reinforcing and perpetuating, or amplifies real or perceived skill differentials under Bayesian learning. For example, an initial IPO success makes the entrepreneur more likely to become a future trusting investor and provide network support for the VC’s future funds. In essence, our theory formalizes and extends the notion

of the “snowball effect” or the “Matthew effect,” in settings in which the initial heterogeneity could come from luck as well as innate differences.⁴ Importantly, because our model mechanism relies on the interaction between fund heterogeneity and endogenous deal flow through assortative matching, it does not predict performance persistence in other delegated investment such as the mutual fund industry, where endogenous deal flow is absent.

Finally, our theory has several practical implications for investors, venture capitalists, and entrepreneurs. First, limited partners that are long-lived in the market and interacting with multiple VC firms should take advantage of the inter-contract incentives in managing agency rent and motivating effort from managers; investors without the power to offer contract hierarchy should still invest in funds based on past performance. Second, fund managers need to go beyond presenting a simple track record to demonstrate superior skill and value-added to investors and entrepreneurs. This is consistent with limited partners’ recent focus on more detailed information about funds such as their internal organization, culture, deal sourcing, etc. (see also Korteweg and Sorensen (2017)). Finally, entrepreneurs choosing which VC to work with should focus not only on the funds’ status and past success, but also on the exact advantages of the funds or characteristics past successes endogenously generate.

Literature — Our paper contributes foremost to the large literature on managerial skill and fund performance. Berk and Green (2004) illustrate that the lack of return persistence in mutual funds does not necessarily imply the absence of skill difference. Garleanu and Pedersen (2015) show that search frictions can lead to a persistent spread in net-of-fee returns. Hochberg, Ljungqvist, and Vissing-Jorgensen (2014) argue that incumbent investors with insider information can hold up managers and extract information rents. Marquez, Nanda, and Yavuz (2015) suggest that the excessive efforts of VCs to manipulate the entrepreneur’s beliefs about his ability also leads to persistence. Acharya, Gottschalg, Hahn, and Kehoe (2013) analyze deal-level data and show that managers with disparate backgrounds add value in different deals. Gompers, Kovner, Lerner, and Scharfstein (2010) argue that perfor-

⁴The “Matthew effect”, originally introduced in the book of Matthew 25:29, is a phenomenon sometimes summarized by the adage that “the rich get richer and the poor get poorer”. See, for example, Azoulay, Stuart, and Wang (2013) and Simcoe and Waguespack (2011).

mance persistence in entrepreneurship can be attributed to entrepreneurs' (perceived) skills. Kacperczyk, Van Nieuwerburgh, and Veldkamp (2014) discuss time-varying fund manager skill.

Different from these papers in which persistent performance are often predicated on managers being inherently heterogeneous (for example, in skills), our paper shows that transitory luck and endogenous deal flow could drive performance persistence. Glode and Green (2011) take a similar agnostic view on the innate skills of managers, and show that in the hedge fund industry, concerns about information spillover gives incumbent investors bargaining power and leads to persistence in excess returns. Hoberg, Kumar, and Prabhala (2017) find that mutual funds outperforming their customized rivals generate future alpha when they face less competition. Our paper differs both in the mechanism and the application. More importantly, we bring together in a general equilibrium model three key players in the private equity market—investors, fund managers, and entrepreneurs. This general equilibrium framework allows us to analyze the distribution of contracts and deal allocations among all funds.

Our paper is thus closely related to and broadly consistent with several empirical studies: Nanda, Samila, and Sorenson (2017) document performance persistence at the investment level and suggest that performance persistence stems from improved deal flows rather than from managers' innate ability. Sorensen (2007) finds that companies funded by more experienced VCs are more likely to go public, and structurally estimates that deal flows (sorting) are twice as important as direct value added by VCs (influence) in explaining the observation. Korteweg and Sorensen (2017) suggest that VC performance is mostly driven by luck. Also related are Hochberg, Ljungqvist, and Lu (2007) and Venugopal and Yerramilli (2017) which provide evidence that an emerging track record of success improves a VCs and angel investors' network position over time, which leads to persistent out-performance. Our model provides a framework to rationalize their findings. In particular, our model predicts both long-term mean-reversion in fund performance and predictability of average performance of other funds observed in Nanda, Samila, and Sorenson (2017), and demonstrates that the importance of deal flow highlighted in Sorensen (2007) does not necessarily rely on

VC heterogeneity.

Our paper further contributes to the growing literature concerning the role of intermediaries. Hellmann and Puri (2000) document that VCs are associated with significant reductions in commercialization time for innovative products; Kortum and Lerner (2000) find that increased VC activity in an industry leads to significantly more innovation; Nanda and Rhodes-Kropf (2016) show that VC investments facilitate riskier experimentation and more innovative start-ups. Our paper complements these studies by demonstrating how past successes lead to heterogeneity across VC funds in facilitating innovation, a la Manso (2011) who argues that the optimal way to motivate innovation is to show tolerance for failure. In so doing we provide theoretical foundations for the phenomena about failure-tolerance and innovation documented in Tian and Wang (2014) and Landier (2005), and about the certification effect of VC investment described in Hsu (2004). Instead of resorting to manager skills, we emphasize the role of fund investors and endogenous deal flows.

From a theoretical perspective, our discussion on hierarchical contracts is related to delegated investment and dynamic agency. Bolton and Scharfstein (1990) and Stiglitz and Weiss (1983) examine funding termination as a way to mitigate managerial incentive problems. Chung, Sensoy, Stern, and Weisbach (2012) show that future fund-raising creates significant incentives for private equity funds to perform well. Our paper extends the discussion to contracting between investors and fund managers, which is important but largely unexplored (Rin, Hellmann, and Puri (2013)).

Although equilibrium contracts in our model exhibit features seen in partial-equilibrium dynamic contracting papers, such as back-loading of pay,⁵ our paper focuses on the performance persistence from the principal's perspective. Moreover, our paper illustrates how intern-contract incentives helps incentive provision, and link an individual's contract and dynamic moral hazard to aggregate market conditions. Embedding the contracting problem in a general equilibrium framework allows us to characterize the distribution of contracts in the economy, and relate luck-induced allocation of contracts and deal flows, which is new to the literature. In this regard, Axelson and Bond (2015) also features homogeneous employees

⁵See, for example, Bolton and Dewatripont (2005), Sannikov (2008), Biais, Mariotti, Rochet, and Villeneuve (2010), and Edmans, Gabaix, Sadzik, and Sannikov (2012).

receiving differential contracts with different utilities. We mainly differ in our emphasis on agents' endogenous opportunity set and associated risk allocation among agents. Therefore, relating to studies on contracting with externalities (e.g., Segal (1999)) in general, our innovation lies in analyzing the incentives channel generated by the limited supply of innovative projects in fund manager markets.

The rest of the paper is organized as follows: Section 2 lays out the basic framework and characterizes the dynamic equilibrium; Section 3 presents model implications; and Section 4 further discusses and extends the model. Finally, Section 5 concludes.

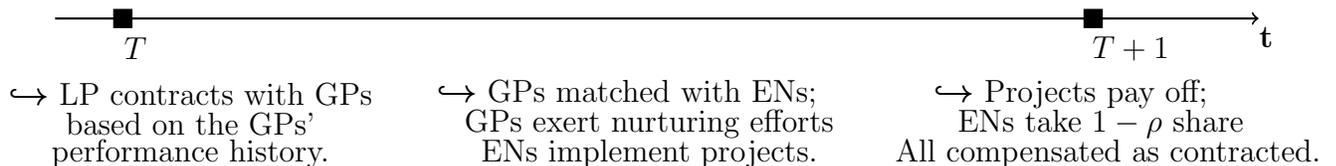
2 Model Setup

To underscore the stark effect of luck in performance persistence, we assume that fund managers are homogeneous in skill and focus on the moral hazard of effort provision. Section 4.3 discusses how differential manager skill can be significantly amplified under dynamic learning by endogenous fund heterogeneity and deal flow that we highlight.

2.1 Dynamic Environment

Time is discrete and infinite, and is labeled by $t = 1, 2, 3, \dots, \infty$. There are three groups of agents: entrepreneurs (EN), venture fund managers (GPs), and investors (LPs). For simplicity, all players are risk-neutral and share the same time discount factor $\beta \in (0, 1)$. Figure 1 provides the time-line, and the sequence of actions that we elaborate below.

Figure 1: Timeline within a period.



First, a unit measure of entrepreneurs (EN) are born in each period, each endowed with a project with observable quality. For simplicity, we assume two types of projects: innovative

projects (I-projects) and conventional projects (C-projects). It is easy to extend our analysis to cases with multiple or a continuum of project types, and our main results are qualitatively unchanged. A fraction $\phi \in (0, 1)$ of entrepreneurs have innovative projects, and the rest have conventional projects. Let \mathcal{E}_t denote the set of all time- t entrepreneurs. Each project requires an investment K and nurturing by venture capitalists. A conventional project succeeds with probability $p_C \in (0, 1)$ and pays off X_C , whereas an I-projects succeeds with probability $p_I < p_C$ and pays off $X_I > X_C$. A project yields $X_f < \min\{X_C, X_I\}$ upon failure, which we normalize to zero. Each entrepreneur lives for one period and then permanently exits the market.⁶

Second, there is an infinite supply of venture capitalists aspiring to become general partners of VC funds (GPs). Hence, unlike Jovanovic and Szentes (2013), our results are not driven by the scarcity of VCs. In each period, after successful fundraising, a GP $g \in G_t$ makes offers and is endogenously matched with an entrepreneur.⁷ They also decide whether or not to incur effort cost $e > 0$ to augment the project's success probability by $\Delta \in \left(0, \frac{1-p_C}{p_C}\right)$, with the upper bound simply reflecting that the augmented success probability cannot exceed one. Our baseline model recognizes that GPs provide value-added services because they screen, improve, and nurture projects, but they do not have differential skills.

In the next period, the fund's investment outcome is realized. The GP pays the entrepreneur and investors, closes the old fund, and raises capital for a new fund. A GP permanently exits the market of venture capital upon failure of fundraising. For simplicity, we assume the GPs contract with entrepreneurs to share the project payoff in fixed proportions $\rho : 1 - \rho$, $\rho \in (0, 1)$.⁸ A GP's strategy therefore involves $\Lambda_g = \mathbb{I}_e$, where \mathbb{I}_e is the effort indicator.

Third, there is one representative investor who can invest in multiple funds each period

⁶In other words, each entrepreneur only interacts with venture capitalists once, which captures reality in a reduced-form. While serial entrepreneurs exist, they are rare and their contracts with financiers are based on individual projects; start-ups not immediately funded are often out-competed by rivals.

⁷For simplicity, we assume one GP contracts with one entrepreneur, to capture that in reality each fund only operates a limited number of projects, reflecting anecdotal evidence that VCs' scarce resource is time and deals (whether evaluation or nurturing) requiring approximately equal amounts of time. Quindlen (2000) and Kaplan and Strömberg (2004) provide more details.

⁸Endogenizing the contract terms between GP and EN in a Nash bargaining game would not alter the main results.

as a limited partner (LP). Readers can think of her as a university endowment, a pension fund, or a family office, etc, with deep pockets to finance all potential projects. We extend the discussion to multiple LPs in Section 4.1 and show LP competition does not qualitatively alter our results. In each period t , she invests in a unit measure of funds, and the set of time- t funds is denoted by F_t . She decides her investment plan $A_t(f)$, $f \in F_t$, which is a mapping from performance history to contracts. Then the time- t set of GPs that are successful at fundraising can be denoted by $G_t = \{g | \exists f \in F_t, \text{ s.t. } g = A_t(f)\}$. For each fund she works with, based on the fund's past performance history, she offers contracts of the form $\Phi_g = \{\sigma_f, \alpha, V_f, V_s\}$. σ_f can be interpreted as management fee and α as the GPs' carried interest. V_S , $S = \{s, f\}$ is g 's promised continuation value given the fund's project outcome and all agents' equilibrium strategies. One can rewrite the contract as $\Phi_g = \{\sigma_f, \sigma_s, V_f, V_s\}$, where σ_s and σ_f are GP g 's cash payments conditional project success and failure respectively, and σ_s is determined by $\alpha \equiv \frac{\sigma_s - \sigma_f}{\rho(X_j - X_f)}$, $j \in \{C, I\}$. Without loss of generality, at any time t if g has not started a fund yet or if he fails to raise capital for a consecutive fund, the offer is denoted by $\Phi_g = 0$.

The continuation value V_S , $S = \{s, f\}$ of a contract comes from future contracts from the LP. For example, if contract Φ_g promises that upon project failure, in the next round GP g receives the same contract Φ_g with probability 20%, a different contract Φ'_g with probability 50%, and is fired with probability 30%, then $V_f = 20\%V_{GP}^{\Phi_g} + 20\%V_{GP}^{\Phi'_g} + 30\% \times 0$, where $V_{GP}^{\Phi_g}$ is GP g 's valuation of contract Φ_g . Using continuation as an incentive is consistent with findings in Chung, Sensoy, Stern, and Weisbach (2012), who find that 40% of manager pay is for indirect compensation from future fund raising. While in standard partial equilibrium contracting environments only the level of continuation value matters, in our general equilibrium model the form of continuation values also matters because contracts interact and the aggregate investment opportunity set is fixed (the distribution of I-projects and C-projects). Since the distribution of all fund contracts are time invariant in a stationary equilibrium, a change in one contract's continuation value does not only change the contract itself, but also indirectly affects other contracts that involves future continuations with the contract in question.

While in our baseline model each fund offers the same ρ to entrepreneurs, fund choice is important for entrepreneurs because GPs in general can influence firm operations, team building, and experimentation styles by setting different contract terms, taking hidden actions, etc. To simplify our analysis, we assume that entrepreneurs, when indifferent in terms of monetary payoff, prefer to be matched with GPs with smaller difference in continuation values $V_s - V_f$. That is to say, given the proposed set of funds F_t^i , EN i 's fund choice is:

$$\Psi_i = \{j | j \in F_t^i, V_s^j - V_f^j \leq V_s^k - V_f^k, \forall k \in F_t^i\}. \quad (1)$$

One interpretation is that $V_s - V_f$ reflects a GP's attitude towards failure. Motivated by Manso (2011), a smaller difference indicates a milder punishment for failure, making GPs more tolerant towards entrepreneurs' initial failures; another interpretation is that such GPs are so well-established that they are no longer sensitive to one project's success or failure, making them better collaborators for a project's long-term goal.⁹ The particular specification of entrepreneurial preference is not crucial to our results, and is only used to break indifference. Alternative specifications are fine as long as it produces the empirically observed pattern that other things being equal, entrepreneurs prefer to be matched with funds with a better track record.

We model project deal flows by two-sided stable matching in each period between projects and the GPs with successful fundraising. As in the Deferred-Acceptance Algorithm (Gale and Shapley (1962)), GPs observe entrepreneurs' types and simultaneously make offers to their top choices. Entrepreneurs reject all but their top choices of funds, and break indifference by randomizing among the funds they equally prefer. Rejected funds then make the next round of offers, and the remaining entrepreneurs again reject all but their top choices. The process goes on until all the funds have projects or all the projects have VC backing.

For ease of exposition, we make two additional assumptions in our baseline model.

⁹One can consider alternative entrepreneurial preferences in reduced forms. For example, they could prefer funds with a better track record because such funds are more salient and better connected with the venture community. Another possibility is that better compensated GPs are more pleasant to work with because they are not close to being terminated and are less stressed. Though outside our model, if the funds bid for projects, GPs with higher agency rent can also afford to bid slightly more, which benefits the ENs. We elaborate on these alternatives later and discuss why they would not change the main results.

Assumption 1.

$$X_{IpI}(1 - \beta p_C) \geq X_{cPc}(1 - \beta p_I) \quad (I\text{-project Superiority}). \quad (2)$$

The first assumption says that I-projects in terms of expected payoff is superior to C-projects by a certain margin. As shown in the proof for Lemma 3, this margin makes GPs prefer I-projects over C-projects. Were the LP able to dictate in the contract whether a GP can only choose an I-project or a C-project, this assumption and equation (1) would not be needed and it would suffice to have $X_{IpI} > X_{cPc}$. However, in practice the LP may have limited information and power to interfere how GPs are matched with projects.

Assumption 2.

$$\phi \leq \frac{p_C(1 + \Delta)}{1 + (p_C - p_I)(1 + \Delta)} \quad (Unicorn Scarcity). \quad (3)$$

The second assumption says I-projects are scarce enough that all new managers are matched with C-projects. It is introduced to simplify our discussion and is relaxed in section 4.2.

2.2 Equilibrium Definition

Following prior studies such as Levin (2003), Jovanovic and Szentes (2013), and Axelson and Bond (2015), we focus on dynamic equilibria that are stationary.

Definition 1. *An equilibrium consists of LP's strategy $\Xi^* \equiv \{\{\Phi_f^*\}_{f \in F_t}, A_t\}_{t=1,2,\dots}$, GPs' strategies $\{\{\Lambda_g^*\}_{g \in G_t}\}_{t=1,2,\dots}$ and entrepreneurs' strategies $\{\{\Psi_i^*\}_{i \in I_t}\}_{t=1,2,\dots}$ such that:*

1. *For each GP $g \in G_t$, conditional on entrepreneurs' funding offer choices $\{\Psi_i^*\}_{i \in I_t}$, LP's contract Φ_g^* and other GPs' strategies $\{\Lambda_{g'}^*\}_{g' \in G_t \setminus g}$, Λ_g^* satisfies:*

$$\Lambda_g^* \in \operatorname{argmax}_{\Lambda_g} E^{\Lambda_g} \{\sigma_S + \beta V_S\}; \quad (4)$$

where σ_S is the GP g 's cash payment, and V_S is the promised value in state $S \in s, f$;

2. For each EN $i \in \mathcal{E}_t$, conditional on GPs' contracts $\{\Phi_f^*\}_{f \in F_t}$, her funding offer choice Ψ_i^* satisfies equation (1);
3. Conditional on GPs' strategies $\{\{\Lambda_g^*\}_{g \in G_t}\}_{t=1,2,\dots}$ and entrepreneurs' strategies $\{\{\Psi_i^*\}_{i \in I_t}\}_{t=1,2,\dots}$, Ξ^* maximizes LP's discounted expected investment profit:

$$\Xi^* \in \operatorname{argmax}_{\Xi} E^{\Xi} \left\{ \sum_{t=1}^{\infty} \beta^t \left[\int_{G_t} (p_S X_S - \sigma_S) df - K \right] \right\}. \quad (5)$$

A *Stationary Equilibrium of Delegated Investment* is an equilibrium such that:

1. The set of fund contracts $\{\Phi_f^*\}$ are time-invariant and non-random;
2. Let $M_t(\Phi, \Phi')$ be the time t measure of funds whose GPs were offered contract Φ in the last period and receive contract Φ' in the current period, then $M_t(\Phi, \Phi')$ is time-invariant for all $\Phi, \Phi' \in \{\Phi_f^*\}$.

In the stationary equilibrium, the aggregate distribution of funds (contracts) is time-invariant and deterministic. The LP finances constant measures of contracts, and receive time-invariant total investment returns.

2.3 Dynamic Equilibrium and Hierarchical Contracts

We are interested in the case in which the LP want to start funds to finance all projects, and wants to motivate effort from the GPs.¹⁰ Solving the equilibrium could be potentially challenging for two reasons. First, the endogenous matching of deal flow depends on terms of all contracts. Second, for each contract, any promised continuation value derives from a distribution of possible future contracts. In aggregate, the transition rates among all types of contracts should be balanced such that the distribution of contracts is stationary.

¹⁰In other words, and as is standard in contract theory, our analysis focuses on parameters satisfying IR constraints for both GPs and LP, and the effort is worthy. In our model, GPs always want to participate since they can run a fund without exerting any effort. LP's IR constraint would hold if $\beta(1 + \Delta)\rho p_C X_C - \frac{1+\Delta}{\Delta}e - K \geq 0$. LP would find it optimal to motivate efforts if $\beta\Delta\rho p_C X_C > \frac{1+\Delta}{\Delta}e$. The model becomes trivial if any of these conditions is violated.

The interaction between terms of all contracts and endogenous fund-project matching is complicated, we therefore first solve a quasi-equilibrium assuming that the LP can allocate projects to funds directly and then show that the project allocation characterized in the quasi-equilibrium is consistent with entrepreneurs' endogenous fund choices. We first postulate certain characteristics of an equilibrium (if it exists), in the absence of which the LP can deviate to propose different contracts or allocation strategies A_t to induce another steady state to be strictly better off. We then derive the optimal contract and the resulting equilibrium based on those characteristics.

Quasi-Equilibrium

In the quasi-equilibrium, we derive the optimal contract assuming that the LP can directly allocate projects to funds. For expositional simplicity, a fund's contract is called a C-contract if it is matched with a C-project, and is an I-contract if matched with an I-project. In the equilibrium there may be many different types of C-contracts or I-contracts, and they may be historical path dependent. Since all projects are financed and in the equilibrium managers always exert effort, the cash flow generated from projects is fixed. Therefore maximizing the LP's payoff is equivalent to minimizing this total payment to GPs. Since all promised value must be paid in future, the total payment to GPs can be written alternatively as:

$$\mathcal{C} \equiv \frac{e}{1-\beta} + \frac{\beta}{1-\beta} V_{GP}^{new} + V_{GP}^{total}. \quad (6)$$

The first term is the total effort expense incurred from now on; the second term is the present value of future payoffs given to the future new entrant GPs; and the third term represents the present value of all rents to the current GPs.

LP can reward successes and punish the failures by choosing the type of contract, and firing or improving the terms of contract. To induce effort, any C-contract must satisfy:

$$\begin{aligned} & \beta[(1+\Delta)p_C(\sigma_f + \rho X_C \alpha_C + V_s^C) + (1-p_C(1+\Delta))(\sigma_f + V_f^C)] - e \\ & \geq \beta[p_C(\sigma_f + \rho X_C \alpha_C + V_s^C) + (1-p_C)(\sigma_f + V_f^C)], \end{aligned} \quad (7)$$

Therefore, fixing V_s^C and V_f^C , the cheapest contract from the LP's perspective satisfies:

$$\alpha_C = \frac{e - \beta\Delta p_C(V_s^C - V_f^C)}{\beta\Delta p_C \rho X_C}. \quad (8)$$

The payoff (agency rent) for a GP under a C-contract is:

$$V_{GP}^C = \beta[(1+\Delta)p_C(\sigma_f^I + \rho X_C \alpha_C + V_s^C) + (1-p_C(1+\Delta))(\sigma_f^C + V_f^C)] - e = \frac{e}{\Delta} + \beta(\sigma_f + V_f^C). \quad (9)$$

Intuitively, for the GP, the value of operating a C-contract fund consists of two components. $\sigma_f + V_f^C \geq 0$ represents the future payoff in the worst case scenario under limited liability, while $\frac{e}{\Delta}$ describes the GP's minimal agency rent. Similarly, the GP's rent under any I-contract is then:

$$V_{GP}^I = \beta[(1+\Delta)p_I(\sigma_f^I + \rho X_I \alpha_I + V_s^I) + (1-(1+\Delta)p_I)(\sigma_f^I + V_f^I)] - e = \frac{e}{\Delta} + \beta(\sigma_f^I + V_f^I). \quad (10)$$

Notice that in our model, the agency rent largely depends on the payoff in the event of failure ($\sigma_f + V_f$), and the minimal agency rent one can reach in one contract is $\frac{e}{\Delta}$. In the quasi-equilibrium, firing might be costly because it suggests that the LP needs to hire new managers in each period, giving agency rent to them without motivating their previous-period effort. Lemma 1 characterizes contract structure in any quasi-equilibrium.

Lemma 1 (General Contracting Structure). *In the quasi-equilibrium:*

1. For any contract Φ , $\sigma_f^\Phi = 0$;
2. For any contract Φ , $V_{GP}^\Phi \in [\frac{e}{\Delta}, \frac{1}{1-\beta} \frac{e}{\Delta}]$;
3. There exists at least one type of contract Φ that fires GP with non-zero chance upon project failure;

The first two results characterize general features for all contracts in any quasi-equilibrium. Result 1 is straightforward in the sense that management fees do not help motivate effort when agents are risk neutral. Result 2 describes the upper and lower bounds for agency rent

in each contract. It turns out that contracts with the maximum agency rent are the ones that promise to renew with the same contract upon failure. The last result describes the existence of a contract that fires managers. Firing is costly in the sense that the LP needs to grant agency rent to more new GPs in each period. Yet in any equilibrium LP still finds it optimal to do so because otherwise agents would never be fired—job-for-life contracts are not optimal.

All results in Lemma 1 hold regardless of project types. The following lemma characterizes the project allocation among different contracts.

Lemma 2 (LP’s Project Allocation). *In the quasi-equilibrium, for any C-contract Φ and I-contract Φ' , $V_{GP}^{\Phi} \leq V_{GP}^{\Phi'}$.*

The need to balance contract transition rates in a stationary equilibrium drives this allocation pattern. Compared to C-projects, I-projects are riskier and have a higher chance to fail. Thus they are more likely to be demoted to contracts with lower agency rent. Granting I-contracts with higher agency rent results in a relatively smaller fraction of high agency contracts in the steady state because there are fewer I-projects. This arrangement therefore lowers the total agency cost.

We next derive the quasi-equilibrium. First, when the LP is very impatient, we have a complete separation of contracts:

Proposition 1 (Quasi-Equilibrium with Parallel Contracts). *When the LP is impatient, that is, $p_I(1 + \Delta) > \beta$, the LP offers a measure $\phi(1 + \Delta)p_I$ of I-contracts to GPs who are recently successful under I-contracts and a measure $\phi(1 - (1 + \Delta)p_I)$ to new GPs, all with terms:*

1. $\alpha^I = \frac{(1 - \beta p_I)e}{\beta \Delta p_I \rho X_I}$;
2. Renewal of the same contract upon project success and payoff X_I ;
3. Permanent termination of the current GP upon project failure.

The LP offers a measure $1 - \phi$ of C-contracts. To be more specific, she offers a measure $(1 - \phi)(1 + \Delta)p_C$ to GPs who are recently successful under C-contracts and a measure $(1 - \phi)(1 - (1 + \Delta)p_C)$ to new GPs, all with terms:

1. $\alpha^C = \frac{(1-\beta p_C)e}{\beta \Delta p_C \rho X_C}$;
2. *Renewal of the same contract upon project success and payoff X_C ;*
3. *Permanent termination of the current GP upon project failure.*

Intuitively, when the discount factor is very low, the future replacement is not very costly for the LP. She prefers to replace the incumbent GPs upon failure regardless. In the equilibrium, providing I- and C-contracts can be viewed as independent contracting problems for the LP.

Now when the LP's discount factor is in a moderate range, i.e., $(1+\Delta)p_I < \beta \leq (1+\Delta)p_C$, a contract hierarchy endogenously emerges.

Proposition 2 (Quasi-Equilibrium with Hierarchical Contracts). *When $(1+\Delta)p_I < \beta \leq (1+\Delta)p_C$, the LP offers a measure ϕ of I-contracts to GPs who are recently successful with terms:*

1. $\alpha^I = \frac{(1-\beta^2 p_I)e}{\beta \Delta p_I \rho X_I}$;
2. *Renewal of the same contract upon project success and payoff X_I ;*
3. *Continued funding under a C-contract upon project failure.*

The LP offers a measure $1 - \phi$ of C-contracts. To be more specific, she offers a measure $(1 - \phi)(1 - (1 + \Delta)p_C)$ of C-contracts to new GPs, $(1 - \phi)(1 - \lambda)(1 + \Delta)p_C$ to GPs who are recently successful under C-contracts, and $\phi(1 - (1 + \Delta)p_I)$ to GPs who recently failed under I-contracts, all with terms:

1. $\alpha^C = \frac{(1-\beta(1+\lambda\beta)p_C)e}{\beta \Delta p_C \rho X_C}$;
2. *Upon project success and payoff X_C , continued funding with an I-contract with probability λ and renewal of the current C-contract with probability $1 - \lambda$;*
3. *Permanent termination of the current GP upon project failure;*

where $\lambda > 0$ solves $\lambda(1 + \Delta)p_C(1 - \phi) = [1 - (1 + \Delta)p_I]\phi$.

With hierarchical contracts, new GPs are offered C-contracts with low agency rent and are promoted to I-contracts with high agency rent upon their success. GPs with I-contract are demoted to C-contract when they fail, and are terminated if they fail under the C-contract. Intuitively, when the discount factor is high $\beta \geq p_I(1 + \Delta)$, the LP cares about the replacement and future GP costs. To save the cost of terminating GPs under I-contracts, the LP downgrades them to operate C-contracts. Promoting successful C-contract GPs to the more attractive I-contracts increases their promised continuation value upon success. Given no I-contract GPs will be kicked out, this promotion and demotion feature redistributes the continuation values among GPs, providing extra incentive for GPs to exert effort.

For model implications, we focus on the case in which the discount factor is in the moderate range stated in Proposition 2. This simplification allows us to understand hierarchical contracts in their simplest forms.¹¹ We relax this assumption and discuss more general hierarchical-contract structures in Section 4.2.

Inter-contract Incentives

Our general equilibrium framework allows us to analyze the interaction among contracts through IC constraints. Suppose now that there are no I-projects. If the LP offers the optimal contract conditional on only C-projects being available and makes zero profit, then she should be indifferent between investing in funds or not. As shown in the model solution, when both conventional and innovative projects are present, the LP is strictly better off by investing in a non-zero measure set of conventional deals. The existence of another type of project enables the LP to redistribute continuation values among different types of contracts, creating extra incentives for GPs to exert effort. In the case of C-projects, the optimal contract features $V_s^C = V_{GP}^C = \frac{\epsilon}{\Delta}$, while the optimal contract with I-projects features $V_s^C = V_{GP}^I > \frac{\epsilon}{\Delta}$, suggesting a lower α_C and strictly positive profit for the C-fund investment. Moreover, it is straightforward to see that even if the LP loses money investing in C-projects, she may be willing to do so when both types of deals are available because

¹¹An alternative assumption that results in two classes of contracts is that the contract can only be contingent on the current period's success or failure. We solve the case in an earlier draft of the paper and finds the results remain the same. When $\beta > (1 + \Delta)p_C$, there could be more than two layers of contracts.

the loss is dominated by the benefit of cost savings on motivating effort.

By analyzing the LP's portfolio choice of contracts, we also link individual contracts to one another and to aggregate market conditions. For example, when ϕ is higher and the economy is more innovative, the likelihood of promotion from a C-contract goes up, but the payout σ_s goes down. The promotion introduces a change in net-fee performance for funds with C-contracts even though the payoff structure $\{p_C, X_C\}$ does not change. The payoff and evolution of each contract also depend on other funds' performance and contracts, which contrasts with the partial-equilibrium contract literature and the literature on tournaments with exogenous rewards.

Assortative Matching of Funds and Projects

Now we verify that the I-contracts described in the quasi-equilibrium would indeed allow the funds to be matched with I-projects. To see this, we note that

$$V_s^I - V_f^I = V_{GP}^I - V_{GP}^C = \frac{e}{\Delta}(1 + \beta) - \frac{e}{\Delta} < \frac{e}{\Delta} = V_s^C - V_f^C. \quad (11)$$

Thus Ens prefer I-contract funds. At the same time, under Assumption 1, I-contract funds also prefer to be matched with I-projects, as the next lemma reveals.

Lemma 3. *Under I-contracts, GPs receive higher agency rent when matched with I-projects.*

If the LP can directly dictate that GPs working with I-contracts do not get paid when matched with C-projects, and those working with C-contracts do not get paid when matched with I-projects, then the assortative matching is trivial. However, in reality it is difficult for the LP to dictate how projects are matched with funds directly. For example, typically when a GP raises the next fund, the payoff from the current fund is not yet final and verifiable, and fundraising largely depends on some interim performance (e.g., Barber and Yasuda (2017)).

Our specification is just one dimension assortative matching could take place. Arguably, besides entrepreneurs' preference for funds more tolerant towards failure, projects and funds could be matched in practice along several other dimensions. For example, past successes may bring visibility to a fund, which makes it easier for entrepreneurs to find; past successes

may enable the GP to invest more in his next fund, which incentivizes his effort better; if we go beyond the risk-neutral setting, past successes may also increase the GP’s wealth and reduce his risk-aversion, allowing him to be better at nurturing I-projects; finally, GPs with recent successes have more extensive entrepreneurial networks which can save new entrepreneurs’ cost in searching for a core team member or resource.

In all these dimensions, I-projects and even C-projects prefer to be matched with the funds that have a better track record. This would also be consistent with the case in which entrepreneurs believe GPs have differential skills and are inferring that from past performance. Given that in our setting, a smaller $V_s - V_f$ indeed corresponds to a better track record, assortative matching as we specified gives equivalent predictions to that based on all these alternative dimensions or fund characteristics. More importantly, taking a contracting approach allows us to directly model the LP-GP interaction and discuss net-of-fee performance, which alternative setups fail to achieve.

Stationary Equilibria

With the assortative matching result described above, it follows immediately that the project allocation proposed by the LP in the quasi-equilibrium is consistent with ENs’ endogenous funding choice.

Proposition 3 (Stationary Equilibria). *The quasi-equilibria characterized in Propositions 1 and 2 are stationary equilibria.*

With this, we discuss key model implications next.

3 Model Implications

3.1 Fund Performance

Our notion of persistence here refers to persistent dispersion across fund performance, which naturally implies persistent out-performance or under-performance relative to the industry average.

Luck-induced Persistence

The conventional interpretation of fund performance persistence is that managers have differential skills because random luck cannot generate persistent out-performance. Our model shows that even though luck is i.i.d., it can have an enduring impact because the LP optimally allocates contracts to reward successful funds with higher continuation values and lower continuation sensitivity to future performance. This in turn enables funds with successful track records to match with innovative projects. Because I-projects have higher expected payoffs, funds with successful track perpetuate their out-performance.

In terms of gross performance, because recently successful GPs are higher in the hierarchy of contracts, they are more likely matched with I-projects. Given that $(1 + \Delta)p_I X_I > (1 + \Delta)p_C X_C$, their expected gross return is higher. A priori, the contract allocation channel may not give us this prediction: Suppose the GP finances the same type of projects regardless of the contract allocation. Then, conditional on the GP's exerting effort, the expected fund gross return is a constant across different funds. This demonstrates that the endogenous deal flow (matching) plays a significant role in generating gross performance persistence.

In terms of net-of-fee performance, when $p_I(X_I + \frac{\beta e}{\Delta}) > p_C(X_C + \frac{e}{\Delta})$, I-contract yields a higher net-of-fee return. To the extent that a successful GP with an I-contract has a higher probability to continue getting I-contracts relative to other GPs, net-of-fee performance dispersion can be persistent.

One critique for skill heterogeneity as an explanation for persistent performance is that the more skilled GPs can charge a higher fee, thus eroding superior returns to the LP. Our theory survives this critique because GPs do not have differential skills, and if they do seek to extract all the surplus, the LP can simply replace them with a GP that has a similar track record, or from the pool of aspiring GPs. GPs are also in infinite supply in our model, and thus competition (or the lack of) is not the driver for persistence as in Hoberg, Kumar, and Prabhala (2017).

Performance Forecasts and Mean Reversion

If innate skill in choosing promising companies or nurturing them were the only source for performance persistence, then the average performance of *other* VC funds investing in similar sorts of deals (investing in the same industry, sharing the same location, etc.) should not predict the focal GP's future out-performance. However, Nanda, Samila, and Sorenson (2017) use *other* funds' performance (in the same industry) as an instrument, and still finds strong predictive power. Neither do they find evidence for inherent differences in the ability to forecast the macro trends. Instead, they find that success rates decline with experience, and initially under-performing GPs do better over time while those that initially outperform decline in the long run.

Their findings are consistent with our model, which predicts initial luck as a driver for performance persistence and long-term mean reversion of fund performance. To see this, let R_I be the expected performance of an I-contract fund and R_C be the expected performance of a C-contract fund. In the equilibrium with hierarchical contracts, $R_I > R_C$ because under I-contracts GPs execute projects more efficiently, and they are endogenously matched with high quality projects. When an I-contract fund succeeds at time t , the GP can raise another round of I-contract funding, and his time $t + 1$ expected performance is R_I . However, since he may fail at time $t + 1$ and may be demoted to a C-contract, his time $t + 2$ expected performance is $(1 + \Delta)p_I R_I + (1 - (1 + \Delta)p_I)R_C < R_I$. This result comes from the fact that *i.i.d.* shocks in subsequent periods also have a persistent effect. In the long run, they gradually offset the positive impact of initial luck.

We further note that heterogeneity in GP skill cannot fully explain why initially under-performing VC funds do better over time, controlling for fund age to account for learning by doing.

Endogenous Heterogeneity and Investment Opportunity Set

Although we have focused on VC investment, it is worth noting that the insight of this paper applies more generally to delegated investments with endogenous deal flow. For example, initial luck can also play a long-lasting role in buyout funds. While investments

in public markets typically do not involve deal flow, our theory potentially applies to the fund of funds (FoFs) where star funds are matched to FoFs. To the extent that a recently successful FoF is not under pressure to outperform in the short-term and can commit capital over the intermediate and long-term horizons, it can be better matched with star underlying funds so as to perpetuate its success.

We have also focused on the heterogeneous continuation contracts GPs get based on performance history, which is only one form of endogenous fund heterogeneity. More broadly, other forms include the opportunity to invest in later stage firms or to syndicate investment or growth in networks (Nanda, Samila, and Sorenson (2017); Hochberg, Ljungqvist, and Lu (2007); Venugopal and Yerramilli (2017)). Endogenous fund heterogeneity can appear not only at fund level but also at partner level. For example, if a GP helps an entrepreneur to succeed, the latter often becomes an investor for a subsequent fund or acquires the GP's other portfolio firms. This gives VC firms that have become known for their past successes better access to future sought-after deals.

Furthermore, deal flows are just one manifestation of funds' differential investment opportunity sets. The essence of our theory is the complementarity between fund heterogeneity that gives some funds privileged positions, and the resulting differential investment opportunity set. In that sense, if a hedge fund or a mutual fund's recent success leads to a superior investment opportunity set, performance persistence can also emerge. For example, a public equity hedge fund may have more profitable strategies that involve higher short-term risks (for example, due to limits of arbitrage, as seen in the case of LTCM). To the extent that a recently successful hedge fund is not concerned with panic withdrawal by investors, their investment opportunity set is enlarged and may potentially exhibit persistent superior performance, even though other hedge funds know similar strategies.

3.2 Motivating Innovation

In our model, top-performing funds are matched with innovative projects. This endogenous matching is motivated by Manso (2011). As Manso (2011) argues, less punishment towards failure motivates innovation. If the agent is not protected against failures, then the

agent may prefer to exploit in order to avoid failures. In our model, higher agency rent is associated with higher continuation value given project failure ($V_f^I = \frac{e}{\Delta} > 0 = V_f^C$). We have shown earlier that $V_s^I - V_f^I = \beta \frac{e}{\Delta} < \frac{e}{\Delta} = V_s^C - V_f^C$, so the difference between success and failure continuation value shrinks, making GPs less sensitive to potential failure. In Manso (2011), failure protection increases the agency rent because less punishment makes motivating effort more costly. We complement by providing a new perspective on failure protection: it may reduce the total agency rent when the LP is contracting with a group of agents.

Our model also predicts that funds with recent success may be more failure tolerant. This finding is consistent with Tian and Wang (2014), who show that VC’s failure tolerance indeed increases following recent investment success or capital infusion. They also document that firms backed by more failure-tolerant VC investors are significantly more innovative, and this result is not driven by other VC characteristics. In a related study, Townsend (2015) provides evidence that negative shocks to some projects affect the strategy for other projects due to the reduction of resources, or the change of risk attitude towards some projects. To complement these empirical studies, we investigate a theoretical mechanism through which termination as punishment can cost a long-lived principal more for incentivizing nurturing effort. While Ederer and Manso (2013) also examine a similar issue, we allow endogenous matching and model the interactions among all three parties—investors, fund managers, and entrepreneurs. We show that using hierarchical contracts could be a solution.

4 Discussion and Extension

4.1 Multiple LPs and Investor Competition

In the real world, there could be multiple LPs competing for funds and potential deals. We now extend the model to analyze the impact of LP competition. We remark that any model that generates net-of-fee returns cannot allow perfect LP competition. Therefore we focus on the symmetric equilibrium under imperfect competition.

Suppose there are N LPs, and because there is in total a unit measure of projects, each

LP can finance a measure $\frac{1}{N}$ of funds. The first stage of the game is modeled as a Cournot competition in which LPs compete for I-projects by determining what fraction of funds they finance with I-contract. In the second stage, each of them chooses the optimal contracting strategy Ξ_i^* as discussed earlier. Let w_i , $i = 1, \dots, N$ be the fraction of I-contract funds each LP finances, then the measure of all I-contract funds becomes $\frac{\sum_{i=1}^N w_i}{N}$. The return for I-contract funds is negatively correlated with the total supply of I-contracts. With the fixed ϕ supply of I-projects, more I-contract funds may suggest less bargaining power against entrepreneurs, or a smaller chance to be matched with a I-project. Because in equilibrium multiple LPs compete for I-projects using I-contracts, the total number of I-contracts would exceed the number of I-projects. To capture in a reduced form the fact that not every I-contract can be matched with an I-project and the LP competition may reduce the share an I-contract manager bargains from the entrepreneur, we assume that upon success the project payoff to I-contract GPs is a function of the total supply of I-contracts $\tilde{X}_I = X_I - \chi \left(\frac{\sum_{i=1}^N w_i}{N} \right)$, where χ is increasing. This simplifies our exposition without altering the results qualitatively.

We now solve the model by backward induction. Since LPs can always choose to finance all projects with C-contract funds, the cost and benefit of I-contract funds are both measured as their cost and benefit increments compared with C-contract funds. Given Proposition 3, in the first stage, for each LP the average cost increment to issue a larger fraction of I-contract is:

$$C_N \equiv \beta \frac{e}{\Delta} - \frac{\beta}{1 - \beta} \frac{e}{\Delta} (1 - (1 + \Delta)p_C), \quad (12)$$

where $\beta \frac{e}{\Delta}$ is the higher agency rent assigned to the current GP, and $\frac{\beta}{1 - \beta} \frac{e}{\Delta} (1 - (1 + \Delta)p_C)$ is the discounted value of saved replacement costs because I-contract does not fire the GP after failure. On the other hand, the average benefit increment of I-contract funds is:

$$P_N \equiv \frac{\beta}{1 - \beta} (1 + \Delta) \rho \left(p_I \left(X_I - \chi \left(\frac{\sum_{i=1}^N w_i}{N} \right) \right) - p_C X_C \right). \quad (13)$$

Now LP's I-contract ratio decision in the first stage becomes a standard Cournot competition problem. One can interpret the average benefit increment of I-contract funds as the I-contract funds sale price and the marginal cost to issue a larger fraction of I-contract funds as the

constant production cost. Since both price and cost are linear functions in w_i , as in standard Cournot competition models, each LP's optimal I-contract ratio can be solved as:

$$w_i = \left(\frac{p_I X_I - p_C X_C}{p_I \chi} - \frac{(1 - \beta) C_N}{\beta(1 + \Delta) \rho p_I \chi} \right) \frac{N}{N + 1}. \quad (14)$$

It is straightforward to see that $w_i > 0$ and as N increases, the provision of I-contract funds increases. That is to say, competition among fund investors encourages more I-contract funds.

4.2 General Hierarchical Contracts

Now we relax both Assumption 2 and restrictions on the discount factor. In this general case, there exist hierarchical contracts and riskier projects are matched with higher-ranked contracts. We define a level 0 contract as a contract that is terminated for sure upon project failure. Similarly, for any strictly positive integer i , a contract is called a level i contract if the GP would be demoted to a level $i - 1$ contract for sure upon project failure.

Proposition 4 (General Hierarchical Contracts). *There exists an equilibrium such that:*

1. *The LP offers a measure of z_i level- i contracts, $i \in \{0, 1, \dots, n - 1\}$. For each i , a fraction $\varphi_i \in [0, 1]$ of level- i contracts are I-contracts;*
2. *New GPs start with a level-0 contract;*
3. *GPs with level-0 contracts would be promoted to level-1 contracts upon project success;*
4. *GPs with level- $1 \leq k \leq n - 1$ contracts receive level- $\min\{k + 1, n - 1\}$ contracts upon project success;*
5. *The size $\{z_i\}$ and risk allocation $\{\varphi_i\}$ satisfy:*

$$z_j \equiv \begin{cases} z_0 & \text{if } j = 0; \\ z_{j-1} \frac{(1+\Delta)((1-\varphi_{j-1})p_C + \varphi_{j-1}p_I)}{1-(1+\Delta)((1-\varphi_j)p_C + \varphi_j p_I)} & \text{if } 0 < j \leq n - 2; \\ v z_{n-2} \frac{(1+\Delta)((1-\varphi_{n-2})p_C + \varphi_{n-2}p_I)}{1-(1+\Delta)((1-\varphi_{n-1})p_C + \varphi_{n-1}p_I)} & \text{if } j = n - 1; \end{cases} \quad (15)$$

and

$$\varphi_j \equiv \begin{cases} 0 & \text{if } j < i; \\ \varphi_i \in (0, 1] & \text{if } j = i; \\ 1 & \text{if } j > i. \end{cases} \quad (16)$$

where the number of levels n , $v \in (0, 1]$ and i are uniquely determined by

$$\sum_{j=0}^{n-1} z_j = 1, \quad (17)$$

and

$$\sum_{j=0}^{n-1} \varphi_j z_j = \phi. \quad (18)$$

Both proposition 1 and 2 are special cases for this general equilibrium. In the equilibrium, from each agent's perspective, hierarchical contracts work in the same way as in standard one-principal-one-agent dynamic contracting models. New GPs are granted contracts with lowest agency rent and will be promoted to contracts with higher agency rent upon success. GPs are demoted to contracts with lower agency rent when they fail, and will be terminated if they fail under the level-0 contract. In our model, since level- k contract essentially allows the GP to fail at least k times before he is terminated, the level of agency rent in our model can be viewed as a way to measure GP's attitude towards failure.

Our analysis on general equilibrium introduces two interesting features. First, in the optimal contract, even though for each contract, the associated agency rent only depends on V_f and is independent of the project type, the contract transition rates connect continuation terms among all contracts in the equilibrium, pushing the LP to match I-projects to contracts with higher levels of agency rent. Successful GPs from the past are endogenously assigned to more innovative projects, and this endogenous opportunity set provides a channel for generating performance persistence without any innate skill difference.

Second, our framework allows one to analyze contract structure. Contract transition rates imply that every downgraded contract upon failure corresponds to an upgraded lower tier contract after success, and this one-to-one correspondence impose a maximum amount

of higher tier contracts that each lower tier contract can support in the equilibrium. z_j in Proposition 4 describes the evolution of relative sizes of different layers of contracts given z_0 . To minimize the discounted agency cost, the LP needs to determine the optimal size z_0 . A large base of level-0 contracts introduces more new managers in each period, while a small size of level-0 contract implies more hierarchies in contracting structure.

4.3 Amplification of Skill Differentials

Thus far, we have focused on the effort provision of GPs, and we assume no skill differential in order to illustrate how endogenous contract allocation and deal flow can generate performance persistence. In reality, there may very well be a dispersion in manager skills and learning in relation to the different types of manager. For simplicity, we shut down the effort provision channel for now. Instead, using a two-period alternative set-up, we illustrate how endogenous fund heterogeneity and deal flows still matter in such settings.

Specifically, in each period, the LP offers contracts to GPs, and deal flows are still determined by the assortative matching of projects to funds. Instead of having GPs provide effort, we now have GPs have heterogeneous skills that are not known ex ante but can be inferred from GPs' track record. To simplify exposition without compromising the intuition and results qualitatively, we keep $p_I = p_C = p$, and take $\beta = 1$, $\rho = 1$, and $\alpha = 1$. We also focus on the case of symmetric learning in order to avoid complicating key intuitions with signaling by managers.

Heterogeneous Skills

For simplicity, GPs are of two types: high-type GPs that can nurture projects with success probability p_h and low-type GPs with probability p_l . The probability that a GP is a high-type is π_0 and let $S_0 = \pi_0 S$. Further assume that with an additional cost z , the high-type can augment the success probability of projects to $1+S$, where $(1+S)p \leq 1$. z could correspond to additional operational funding, or costly initial user acquisitions done by the GP. A low-type can simply use z to enjoy perks. Finally, we assume $p\Delta S_0[\phi X_I + (1-\phi)X_C] < z < p\Delta S_0 X_I$ to ensure that paying the additional z to potentially augment success probability is not

worthwhile when funds are randomly matched with projects, but could be if a fund gets I-project for sure.

In the first period, all funds appear the same, the projects are randomly matched to GPs, and the LP offers the same contracts to GPs. Because ϕ is sufficiently small, all funds get contracts without the additional z . There is a measure of p projects that are successful, and all corresponding GPs are perceived to be of high-type with probability $\pi_{1s} = \pi_0 \frac{1+S}{1+S_0}$ through Bayesian inference. Failed funds are perceived to be of high-type with probability $\pi_{1f} = \pi_0 \frac{1-p(1+S)}{1-p(1+S_0)}$. Let $S_{1s} = \pi_{1s}S$ and $S_{1f} = \pi_{1f}S$.

In period $t = 1$, without contract hierarchy and deal flows, the LP again gives contracts without the additional z . The expected performance of the recently successful fund is higher than that of a recently failed fund by:

$$D_o = pS_{1s}[\phi X_I + (1 - \phi)X_C] - pS_{1f}[\phi X_I + (1 - \phi)X_C] = p[\phi X_I + (1 - \phi)X_C](S_{1s} - S_{1f}). \quad (19)$$

Clearly, the recently successful GPs continue to out-perform in expectation, due to superior skills. What we emphasize is that endogenous contract allocation and deal flow can amplify this differential. This is important because even when the skill differential vanishes, i.e., $S_{1s} - S_{1f}$, recently successful funds can still outperform, as we show next.

Suppose we allow endogenous deal flow only, I-projects rationally seek recently successful funds because the posterior on their managers' skill/type is higher. The measure of successful funds by the Law of Large Numbers is $p > \phi$. When unicorns are scarce (Assumption 2), the probability of each previously successful fund getting an I-project is $\frac{\phi}{p}$, while previously failed GPs are only matched with C-projects. The expected performance of the recently successful fund is higher than that of a recently failed fund by:

$$D_d = pS_{1s} \left[\frac{\phi}{p} X_I + \frac{p - \phi}{p} X_C \right] - pS_{1f} X_C \rightarrow \phi S_0 (X_I - X_C). \quad (20)$$

where the limit is taken as the type difference converges to zero (no learning).

Now if we only allow endogenous future contracts, the LP rationally gives contracts without z to recently failed funds, but gives contracts with the additional z to recently

successful funds if it is profitable in expectation. If a recently successful fund receives an I-contract, its expected performance is higher than that of a recently failed fund by:

$$\begin{aligned}
D_c &= pS_{1s}(1 + \Delta)[\phi X_I + (1 - \phi)X_C] - z - pS_{1f}[\phi X_I + (1 - \phi)X_C] \\
&= p[\phi X_I + (1 - \phi)X_C](S_{1s} - S_{1f}) + pS_{1s}\Delta[\phi X_I + (1 - \phi)X_C] - z
\end{aligned} \tag{21}$$

We note that for S_{1s} big enough, the second term could be positive, indicating performance persistence with endogenous capital and contract alone.

Finally, if we endogenize both contract allocation and deal flow, the LP gives ϕ measure of contracts with additional z to recently successful funds. Then, all projects prefer these funds, who then would be matched with I-projects and some C-projects. The expected performance of the recently successful fund is higher than that of a recently failed fund by:

$$\begin{aligned}
D_{c\&d} &= pS_{1s}(1 + \Delta) \left[\frac{\phi}{p}X_I + \frac{p-\phi}{p}X_C \right] - \frac{\phi}{p}z - pS_{1f}X_C \\
&\rightarrow \phi(X_I - X_C) + \Delta S_0[\phi X_I + (p - \phi)X_C] - \frac{\phi}{p}z
\end{aligned} \tag{22}$$

where the limit is again taken as the divergence of types goes to zero.

We first note that $D_d > D_o$ obviously, and D_c is bounded below by D_o . Both contract allocation and deal flows amplify persistent performance dispersion. There is also a compounding amplification when both contract allocation and deal flows are endogenous because $D_{c\&d} > \max\{D_c, D_d\}$ for all values. As we take the limit that type differential goes to zero, i.e., $\pi_0 \rightarrow 1$, we have $D_o \rightarrow 0$, $D_d \rightarrow \phi(X_I - X_C)$, D_c becomes negative, but $D_{c\&d}$ remains the biggest. When we further take the limit that I-projects are very scarce, i.e., $\phi \rightarrow 0$, we only observe persistent dispersion in performance when both contract allocation and endogenous assortative matching are present. Therefore, a very small dispersion in skills can lead to significant persistence and dispersion in performance, and when I-projects are very scarce ($\phi \rightarrow 0$), the amplification is one magnitude higher than that with either endogenous cash flow only or endogenous deal flow only ($D_{c\&d} \rightarrow \Delta S p X_C > 0$ where as D_c becomes negative and $D_d \rightarrow 0$).

Reputation and Behavioral Interpretation

The evolution of beliefs in the above analysis can be interpreted as GP reputation. GPs with better reputation naturally are less subject to short-term interim performance, and take actions that are beneficial over the long-term (e.g., Barber and Yasuda (2017)).

The above analysis also applies when there is no real skill differential, but only the perception of it.¹² Suppose $S = 0$, but both the LP and ENs believe $S > 0$, then we still observe $D_{c\&d} > 0$. If the LP believes $S > 0$, and ENs know $S = 0$ but understand the LP's belief, then the ENs would anticipate the contract evolution and would rationally opt for the recently successful funds if they have I-projects. The case where ENs believe $S > 0$ and the LP is rational is similar. In short, even if there is no skill differential, but either the LP or the ENs perceive luck as a superior skill, the endogenous fund heterogeneity and deal flows can still generate performance persistence and predictable dispersion. The conclusion shares the spirit of Gompers, Kovner, Lerner, and Scharfstein (2010), but concerns perceived fund manager skills rather than perceived entrepreneur skills.

5 Conclusion

We present a dynamic model of delegated investment that produces performance persistence without skill differences among managers. Contracts with continuation values less sensitive to current performance exhibit strong complementarity with innovative projects, which thus endogenously flow to recently successful managers through assortative matching and the incentivization of managerial effort through continuation value. The main intuition applies to other forms of endogenous fund heterogeneity and project matching that affect future opportunity sets of investment. Consistent with empirical findings, our model predicts that venture funds that persistently outperform attract quality innovative projects with seemingly less favorable contract terms. The model further predicts an “incumbent bias” in allocating continuation contracts to fund managers, mean reversion of funds’ long-term

¹²Nanda, Samila, and Sorenson (2017) also suggest that persistent performance appears to stem from initial differences in success creating beliefs about ability that persist as investors, entrepreneurs and others act on those beliefs.

performance, inter-contract incentives for effort provision, and amplification of small skill differences.

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Appendix

Proofs

Proof of Lemma 1

Proof. Result 1: For any contract $\Phi = \{\sigma_f > 0, \alpha_s, V_f, V_s\}$, the LP can replace the contract with $\Phi' = \{0, \alpha_s, V_f, V_s\}$. The change keeps contract transitions balanced and lowers the associated agency rent. Without loss of generality, we only focus on contracts with $\sigma_f = 0$.

Result 2: The minimum of agency rent is straightforward by setting $V_f = 0$. Suppose the maximum value of agency rent in the quasi-equilibrium is \bar{V}_{GP} , then $\frac{e}{\Delta} + \beta\bar{V}_{GP} \geq \frac{e}{\Delta} + \beta V_f, \forall V_f$. Thus $\bar{V}_{GP} \leq \frac{e}{\Delta} + \beta\bar{V}_{GP}$, and $\bar{V}_{GP} \leq \frac{1}{1-\beta} \frac{e}{\Delta}$.

Result 3: Suppose in the quasi-equilibrium there is no termination, and let the minimum value of contract agency rent in the quasi-equilibrium to be \underline{V}_{GP} . Then $\frac{e}{\Delta} + \beta\underline{V}_{GP} \leq \frac{e}{\Delta} + \beta V_f, \forall V_f$. Thus $\underline{V}_{GP} \geq \frac{e}{\Delta} + \beta\underline{V}_{GP}$, and $\underline{V}_{GP} \geq \frac{1}{1-\beta} \frac{e}{\Delta}$. From result 2, $V_{GP} \leq \frac{1}{1-\beta} \frac{e}{\Delta} \leq \underline{V}_{GP}$, so all contracts share the same agency rent $\frac{1}{1-\beta} \frac{e}{\Delta}$.

Now consider alternative arrangement: Every fund manager is fired upon failure. Now all contracts share the same agency rent $\frac{e}{\Delta}$. The total discounted agency rent becomes:

$$\begin{aligned} & \frac{e}{\Delta} + \frac{\beta}{1-\beta} [\phi(1 - (1+\Delta)p_I) + (1-\phi)(1 - (1+\Delta)p_C)] \frac{e}{\Delta} \\ & < (1 + \frac{\beta}{1-\beta}) \frac{e}{\Delta} \\ & = \frac{1}{1-\beta} \frac{e}{\Delta} \end{aligned} \tag{23}$$

This alternative arrangement generates a lower discounted agency rent and it is obviously a steady state. Thus any quasi-equilibrium must involve termination. \square

Proof of Lemma 2

Proof. Suppose it is not the case, then one can find a measure of ω_C C-contract $\Phi_C = \{0, \alpha_s^{\Phi_C}, V_s^{\Phi_C}, V_f^{\Phi_C}\}$ and a measure of ω_I I-contract $\Phi_I = \{0, \alpha_s^{\Phi_I}, V_s^{\Phi_I}, V_f^{\Phi_I}\}$ such that $\omega \equiv \min\{\omega_C, \omega_I\} > 0$ and $V_{GP}^{\Phi_C} > V_{GP}^{\Phi_I}$ (that is to say, $V_f^{\Phi_C} > V_f^{\Phi_I}$).

We then show that the LP finds it beneficial to deviate to another steady state. Consider the following deviation: Replace a measure of ω contract Φ_C with contract Φ'_C , and replace a measure of $\frac{1-(1+\Delta)p_C}{1-(1+\Delta)p_I} \omega$ contract Φ_I with contract Φ'_I . $\Phi'_C = \{0, \alpha_s^{\Phi'_C}, V_s^{\Phi_C}, V_f^{\Phi_I}\}$, $\Phi'_I = \{0, \alpha_s^{\Phi'_I}, V_s^{\Phi_I}, V_f^{\Phi_C}\}$, where $\alpha_s^{\Phi'_C}$ and $\alpha_s^{\Phi'_I}$ are pinned down by corresponding IC constraints.

Since both $V_s^{\Phi_C}$ and $V_s^{\Phi_I}$ remain unchanged after deviation, the contract transition upon project success remains the same. When the project fails, the original contract allocation transition is $\omega(1-(1+\Delta)p_C)V_f^{\Phi_C} +$

$\frac{1-(1+\Delta)p_C}{1-(1+\Delta)p_I}\omega(1-(1+\Delta)p_I)V_f^{\Phi_I}$. If the LP deviates, then the contracts transition when the project fails is

$$\begin{aligned} & \omega(1-(1+\Delta)p_C)V_f^{\Phi_I} + \frac{1-(1+\Delta)p_C}{1-(1+\Delta)p_I}\omega(1-(1+\Delta)p_I)V_f^{\Phi_C} \\ &= \frac{1-(1+\Delta)p_C}{1-(1+\Delta)p_I}\omega(1-(1+\Delta)p_I)V_f^{\Phi_I} + \omega(1-(1+\Delta)p_C)V_f^{\Phi_C}. \end{aligned}$$

So the deviation is still a steady state.¹³ However, the agency rent is lower:

$$\begin{aligned} & \omega V_{GP}^{\Phi_I} + \frac{1-(1+\Delta)p_C}{1-(1+\Delta)p_I}\omega V_{GP}^{\Phi_C} \\ &= \omega V_{GP}^{\Phi_C} + \frac{1-(1+\Delta)p_C}{1-(1+\Delta)p_I}\omega V_{GP}^{\Phi_I} - \frac{(1+\Delta)(p_C-p_I)}{1-(1+\Delta)p_I}\omega(V_{GP}^{\Phi_C} - V_{GP}^{\Phi_I}) \\ &< \omega V_{GP}^{\Phi_C} + \frac{1-(1+\Delta)p_C}{1-(1+\Delta)p_I}\omega V_{GP}^{\Phi_I}. \end{aligned}$$

□

Proof of Proposition 1

Proof. Proposition 1 is a special case of Proposition 4, therefore we refer the readers to the proof of Proposition 4 first. Given that, all we need to show here is that when $\beta < (1+\Delta)p_I$, $\{n=1, z_0=1, \varphi_0=\phi, \varphi_1=1\}$ is the optimal solution to minimize the total discounted agency cost characterized in equation (27).

For type $i \in \{C, I\}$ contract, if the LP offers a level-1 contract, then the associated agency rent is $\frac{e}{\Delta}(1+\beta)$. If the LP offers a level-0 contract, then the associated agency rent is

$$\frac{e}{\Delta} + \frac{\beta}{1-\beta}(1-(1+\Delta)p_i)\frac{e}{\Delta},$$

where the first term is the agency rent for the incumbent GP and the second term is the discounted value of agency rent granted to new GPs. The LP would prefer level-0 contract if and only if $\beta \leq (1+\Delta)p_i$. When $\beta < (1+\Delta)p_I$, the LP prefers level-0 C-contracts and level-0 I-contracts. □

Proof of Proposition 2

Proof. Proposition 2 is also a special cases of Proposition 4. With Proposition 4, all we need to show is that under Assumption 2 and the fact $(1+\Delta)p_I < \beta \leq (1+\Delta)p_C$, $\{n=2, z_0=1-\phi, z_1=\phi, \varphi_0=0, \varphi_1=1\}$ is the optimal solution to minimize the total discounted agency cost characterized in equation (27).

For type $i \in \{C, I\}$ contract, if the LP offers a level-1 contract, then the associated agency rent is $\frac{e}{\Delta}(1+\beta)$. If the LP offers a level-0 contract, then the associated agency rent is

$$\frac{e}{\Delta} + \frac{\beta}{1-\beta}(1-(1+\Delta)p_i)\frac{e}{\Delta},$$

¹³To be rigorous, when we change contracts, we also change the associated transition terms V_s or V_f in some contracts.

where the first term is the agency rent for the incumbent GP and the second term is the discounted value of agency rent granted to new GPs. The LP would prefer level-0 contract if and only if $\beta \leq (1 + \Delta)p_i$. Since $(1 + \Delta)p_I < \beta \leq (1 + \Delta)p_C$, the LP prefers level-0 C-contracts and level-1 I-contracts. Assumption 2 ensures that level-0 C-contracts and level-1 I-contracts satisfy the transition rate constraint characterized by equation (26). \square

Proof of Lemma 3

Proof. We show that GPs with I-contracts always prefer I-projects to C-projects. To be more specific, we show that

$$\begin{aligned}
V_{GP}^I &= \beta[(1 + \Delta)p_I(\rho X_I \alpha_I + V_s^I) + (1 - p_I(1 + \Delta))V_f^I] - e \\
&= \beta[(1 + \Delta)p_I(\rho X_I \alpha_I + V_s^I - V_f^I) + V_f^I] - e \\
&\geq \beta[(1 + \Delta)p_C(\rho X_C \alpha_I + V_s^I) + (1 - p_C(1 + \Delta))V_f^I] - e \\
&= \beta[(1 + \Delta)p_C(\rho X_C \alpha_I + V_s^I - V_f^I) + V_f^I] - e,
\end{aligned} \tag{24}$$

and

$$\begin{aligned}
\beta[p_I(\rho X_I \alpha_I + V_s^I) + (1 - p_I)V_f^I] &= \beta[p_I(\rho X_I \alpha_I + V_s^I - V_f^I) + V_f^I] \\
&\geq \beta[p_C(\rho X_C \alpha_I + V_s^I) + (1 - p_C)V_f^I] \\
&= \beta[p_C(\rho X_C \alpha_I + V_s^I - V_f^I) + V_f^I].
\end{aligned} \tag{25}$$

Equation (24) suggests that conditional on exerting effort, GPs with I-contracts always prefer I-projects to C-projects. Equation (25) suggests that conditional on not exerting effort, GPs with I-contracts always prefer I-projects to C-projects. With I-contracts, GPs always prefer to exert effort when they are matched with I-projects, thus exerting effort and matched with I-projects always dominates C-projects.

Both equation (24) and (25) are equivalent to $p_I(\rho X_I \alpha_I + V_s^I - V_f^I) \geq p_C(\rho X_C \alpha_I + V_s^I - V_f^I)$. Then:

$$\begin{aligned}
& p_I(\rho X_I \alpha_I + V_s^I - V_f^I) - p_C(\rho X_C \alpha_I + V_s^I - V_f^I) \\
&= \frac{e}{\beta \Delta p_I X_I} (p_I X_I - p_C X_C) - (p_C - p_I + p_I - p_C \frac{X_C}{X_I})(V_s^I - V_f^I) \\
&\geq \frac{e}{\beta \Delta p_I X_I} (p_I X_I - p_C X_C) - p_C \frac{X_I - X_C}{X_I} \frac{e}{\Delta} \\
&= \frac{e}{\beta \Delta p_I X_I} (p_I X_I - p_C X_C - \beta p_I p_C (X_I - X_C)).
\end{aligned}$$

Given Assumption 1, we have

$$\begin{aligned}
& \frac{e}{\beta \Delta p_I X_I} (p_I X_I - p_C X_C - \beta p_I p_C (X_I - X_C)) \\
& \geq \frac{e X_C}{\beta \Delta p_I X_I} \left(p_I \frac{p_C - \beta p_I p_C}{p_I - \beta p_I p_C} - p_C - \beta p_I p_C \frac{p_C - p_I}{p_I - \beta p_I p_C} \right) \\
& = \frac{e X_C}{\beta \Delta p_I X_I} \frac{p_I p_C - \beta p_I^2 p_C - p_I p_C + \beta p_I p_C^2 - \beta p_I p_C^2 + \beta p_I^2 p_C}{p_I - \beta p_I p_C} \\
& = 0.
\end{aligned}$$

So GPs with I-contracts prefer I-projects and ENs with I-projects prefer funds with I-contracts (recall the tie-breaking preference for smaller $V_s - V_f$ ENs have). The project allocation in the quasi-equilibrium is consistent with GPs and ENs matching preference. \square

Proof of Proposition 3

Proof. By construction, for any level- i contract, we have $V_s^{\Phi^i} - V_f^{\Phi^i} = \beta^i \frac{e}{\Delta}$. Thus ENs always prefer funds with higher level contracts, because the monetary payoffs they get are the same. Also from Lemma 3 I-contract funds prefer I-projects, thus all I-contract would be matched with I-project. \square

Proof of Proposition 4

Proof. We will show the existence of the corresponding quasi-equilibrium. From the construction of level- i contracts it is straightforward to see that the quasi-equilibrium is consistent with Ens endogenous funding offer choices.

Step 1: Contract Decomposition

- (i) First of all, for any contract $\Phi = \{0, \alpha_s^\Phi, V_s^\Phi, V_f^\Phi\}$ in the quasi-equilibrium, if Φ^1 is a term in V_s^Φ and Φ^2 is a term in V_f^Φ , then $V_{GP}^{\Phi^1} \geq V_{GP}^{\Phi^2}$. Otherwise the LP can be better off by exchanging some Φ^1 in V_s with some Φ^2 in V_f .
- (ii) We then focus on contracts with termination terms. For any contract $\Phi = \{0, \alpha_s^\Phi, V_s^\Phi, V_f^\Phi\}$ that involves termination, since V_s^Φ does not affect the agency rent and termination is costly. If there is a termination term in V_s^Φ , then the LP can always be better off by replacing the termination term with the corresponding contract offered to new managers. Hence in the quasi-equilibrium termination terms are part of the promised value V_f^Φ upon failure. One can always rewrite $V_f^\Phi = (1 - \gamma_0^\Phi) V_f^{\Phi'} + \gamma_0^\Phi \times 0$, where γ_0^Φ is the conditional probability of termination, and $V_f^{\Phi'}$ is a conditional distribution of all non-termination terms in V_f^Φ conditional on no termination happens. Now consider two contracts $\Phi_0 = \{0, \alpha_s^{\Phi_0}, V_s^\Phi, 0\}$ and $\Phi'_0 = \{0, \alpha_s^{\Phi'_0}, V_s^\Phi, V_f^{\Phi'_0}\}$, where $\alpha_s^{\Phi_0}$ and $\alpha_s^{\Phi'_0}$ are determined by corresponding IC constraints. The contract Φ can be decomposed into a fraction γ_0^Φ of contract Φ_0 and a fraction $1 - \gamma_0^\Phi$ of contract Φ'_0 . $\Phi = \gamma_0^\Phi \Phi_0 + (1 - \gamma_0^\Phi) \Phi'_0$, and it is easy to verify that $\alpha_s^\Phi = \gamma_0^\Phi \alpha_s^{\Phi_0} + (1 - \gamma_0^\Phi) \alpha_s^{\Phi'_0}$. Then any contract with termination terms can be transferred to a contract that faces termination for sure upon project failure.

(iii) We now discuss the term V_f^Φ . Given the discussion above, there are level-0 contracts in the quasi-equilibrium and they are the ones with the lowest agency rent and terminates if and only if the project fails. For any contract $\Phi = \{0, \alpha_s, V_s, V_f\}$ that has level-0 contracts as part of V_f terms, the contract can be decomposed into : $\Phi = \gamma_0^\Phi \times \Phi_0 + \gamma_1^\Phi \Phi_1 + (1 - \gamma_0^\Phi - \gamma_1^\Phi) \Phi'_1$, where $\Phi_0 = \{0, \alpha_s^{\Phi_0}, V_s^\Phi, 0\}$ are level-0 contracts, $\Phi_1 = \{0, \alpha_s^{\Phi_1}, V_s^\Phi, \Phi_0\}$ are level-1 contract that that assigns a level-0 contract for sure upon a project failure, and $\Phi'_1 = \{0, \alpha_s^{\Phi'_1}, V_s^\Phi, V_f^{\Phi'_1}\}$ as the remaining part. Similarly, for any positive integer n , the contract Φ can be further decomposed as $\Phi = \sum_{j=0}^n \gamma_j \Phi_j + (1 - \sum_{j=0}^n \gamma_j) \Phi'_n$. By repeating this decomposition one can construct higher levels contracts.

Given our discussion in (i), if there is a termination term in V_s , then $V_f = 0$. Thus if a contract would be terminated sometime in the future, then it must be a level 0 contract before the termination. If a contract is a level 0 contract, then it is either a new contract or was a level 0 or 1 contract before. Repeating this one can show that if a contract is a level j contract, then it is either a new contract or was a level j or $j + 1$ contract before. Thus for any contract Φ , if V_f^Φ cannot be decomposed as a mixture of contracts with finite levels, then there exists a contract term in V_f^Φ that never fires GPs. As we shown in Lemma 1 result 3, no such contract terms would exist in the quasi-equilibrium, otherwise the LP always want to replace them with level-0 contracts.

- (iv) Similar to the discussion on V_f^Φ , terms in V_s^Φ can also be decomposed as a mixture of contracts with finite levels.
- (v) Let Φ_j^i be a contract that renewal with a level- i contract upon project success and renewal to a level- j contract if the project fails (let level- -1 contract represents firing decision). Our discussion on both V_f^Φ and V_s^Φ suggests that any contract in the quasi-equilibrium can be decomposed as a mixture of contracts $\{\Phi_j^i\}$. Without loss of generosity, any quasi-equilibrium can be rewritten as a quasi-equilibrium with level contracts $\{\Phi_j^i\}$. Also notice that for any contract Φ_j^i in the quasi-equilibrium, $i \geq j$.

Step 2: Contracts for New GPs

We then show that if a contract Φ is offered to new GP, then $V_f^\Phi = 0$. In other words, it must be a level-0 contract.

If it is not the case, then there exists some new manager offered a contract $\Phi = \{0, \alpha_s^\Phi, V_s^\Phi, V_f^\Phi > 0\}$. In the stationary quasi-equilibrium, in each period there would be a fund manager receives such contract. The associated discounted agency rent is

$$\sum_{i=0}^{\infty} \beta^i \left(\frac{e}{\Delta} + \beta V_f^\Phi \right) = \frac{1}{1 - \beta} \left(\frac{e}{\Delta} + \beta V_f^\Phi \right).$$

Every new manager is a replacement for one fired manager. For the new manager with contract Φ , let the corresponding termination contract be $\Phi_0 = \{0, \alpha_s^{\Phi_0}, V_s^{\Phi_0}, 0\}$. Here $V_f^{\Phi_0} = 0$ as we showed in step 1. Then consider the following deviation: replacing new manager's contract Φ with a contract $\Phi' = \{0, \alpha_s^\Phi, V_s^\Phi, 0\}$, and changing the termination contract Φ_0 to $\Phi'_0 = \{0, \alpha_s^{\Phi_0}, V_s^{\Phi_0}, V_f^\Phi\}$.

The deviation directly increases the total agency rent for current managers by $V_{GP}^{\Phi'_0} - V_{GP}^{\Phi_0} = \beta V_f^\Phi$. It may

also indirectly increases the total agency rent for current managers because Φ_0 (and Φ'_0 in the deviation) may be part of continuation value upon project failure in some other contract. If Φ_0 comes from the promised continuation value upon project success in some other contract, then the deviation only changes the corresponding α_s in that contract but not its associated agency rent. If it comes from the promised continuation value upon project failure in some other contract, then by definition it is a level 1 contract Φ_1 , and the corresponding agency rent increment is $\beta(V_{GP}^{\Phi'_0} - V_{GP}^{\Phi_0}) = \beta^2 V_f^\Phi$. Similarly, if Φ_1 is a term of the promised continuation value upon project failure in some other contract, then it is a level 2 contract and the corresponding agency rent increment is $\beta^2(V_{GP}^{\Phi'_0} - V_{GP}^{\Phi_0}) = \beta^3 V_f^\Phi$. If there are n levels of contracts in the quasi-equilibrium, then for the deviation, the maximum value of agency rent increment for the V_{GP}^{total} part is then

$$(\beta + \dots + \beta^n) V_f^\Phi < \frac{\beta}{1-\beta} V_f^\Phi.$$

On the other hand, given the deviation, the saving of discounted agency rent from new managers is $\frac{\beta}{1-\beta} V_f^\Phi$. Thus the LP has incentive to deviate.

Step 3: Hierarchy Structure

Now, suppose in the quasi-equilibrium, the size of level- i contract is z_i , and a fraction $\varphi_i \in [0, 1]$ of those contracts are I-projects. The size and structure contracts to new GPs are also determined by z_0 and φ_0 . Remember that for any level- i contract Φ_{i-1}^j , $j \geq i-1$. Thus for any level- i contracts, by definition all failed funds will be either demoted or fired, so every demoted level- $i+1$ contract must replace a level- i contract that are promoted:

$$z_{i+1}(1 - (1 + \Delta)(\varphi_{i+1} p_I + (1 - \varphi_{i+1}) p_C)) \leq z_i(1 + \Delta)(\varphi_i p_I + (1 - \varphi_i) p_C). \quad (26)$$

The left hand side of this inequality is the size of level- $i+1$ contract that will be demoted to level- i , and the right hand side of this inequality is the maximum size of level- i contract that will be promoted. This inequality puts a natural constraint on the size ratio between level- i and level- $i+1$ contracts.

We then show that if there n levels of contracts in the quasi-equilibrium, then the constraint is always binding for all levels except the highest one ($i+1 = n-1$). Suppose it is not the case, then without loss of generality, let level- j be the lowest level that the constraint is not binding. Define $\omega_j \equiv z_j(1 + \Delta)(\varphi_j p_I + (1 - \varphi_j) p_C) - z_{j+1}(1 - (1 + \Delta)(\varphi_{j+1} p_I + (1 - \varphi_{j+1}) p_C)) > 0$. Since all contracts below level- j satisfy corresponding constraints, in the steady state all contracts above (and include) level- $j+1$ as a whole group generates a measure $z_{j+1}(1 - (1 + \Delta)(\varphi_{j+1} p_I + (1 - \varphi_{j+1}) p_C))$ of outflow to level- j contracts and receives a measure of $z_j(1 + \Delta)(\varphi_j p_I + (1 - \varphi_j) p_C) - \omega_j$ inflow from level- j contract upon project success. Now consider the following deviation: for all contracts above (and include) level- $j+1$, randomly replace a fraction $\frac{\omega_j}{z_{j+1}(1 - (1 + \Delta)(\varphi_{j+1} p_I + (1 - \varphi_{j+1}) p_C))}$ of them with level- $i+1$ contract. Those level- $i+1$ contracts would be renewed with the same level- $j+1$ contract upon project success and would be demoted to level- j contracts upon project failure. On the other hand, randomly choose the same fraction $\frac{\omega_j}{z_{j+1}(1 - (1 + \Delta)(\varphi_{j+1} p_I + (1 - \varphi_{j+1}) p_C))}$ of level- j contracts, change their corresponding V_s terms to those new level- $j+1$ contracts. It is easy to verify that after the deviation it is still a steady state. For all levels except the highest one ($j+1 = n-1$), the deviation changes at least some higher level contracts ($j+2$ or above) to level- $j+1$ contracts, and lowers

the agency rent for incumbent contracts. Thus the size ratio constraints should be binding for all levels except the highest one ($i + 1 = n - 1$).

From Lemma 2 we know that in the quasi-equilibrium, the distribution φ_i is monotonic increasing. Then given z_0 , in the quasi-equilibrium:

$$z_j \equiv \begin{cases} z_0 & \text{if } j = 0; \\ z_{j-1} \frac{(1+\Delta)((1-\varphi_{j-1})p_C + \varphi_{j-1}p_I)}{1-(1+\Delta)((1-\varphi_j)p_C + \varphi_j p_I)} & \text{if } 0 < j \leq n-2; \\ v z_{n-2} \frac{(1+\Delta)((1-\varphi_{n-2})p_C + \varphi_{n-2}p_I)}{1-(1+\Delta)((1-\varphi_{n-1})p_C + \varphi_{n-1}p_I)} & \text{if } j = n-1; \end{cases}$$

and

$$\varphi_j \equiv \begin{cases} 0 & \text{if } j < i; \\ \varphi_i \in (0, 1] & \text{if } j = i; \\ 1 & \text{if } j > i. \end{cases}$$

where the number of levels n , $v \in (0, 1]$ and i are uniquely determined by

$$\sum_{j=0}^{n-1} z_j = 1,$$

and

$$\sum_{j=0}^{n-1} \varphi_j z_j = \phi.$$

Step 4: Existence

We now show the existence of optimal z_0 . First of all, let $M_0 = 1$ and $M_i = \frac{z_i}{z_0}$ for $\forall i \geq 1$. One can rewrite $z_0 = \frac{1}{\sum_{j=0}^{n-1} M_j}$. The total discounted agency rent is

$$\frac{\sum_{j=0}^{n-1} M_j \sum_{k=0}^j \beta^k}{\sum_{j=0}^{n-1} M_j} \frac{e}{\Delta} + \frac{\frac{\beta}{1-\beta}(1 - (1+\Delta)((1-\varphi_0)p_C + \varphi_0 p_I))}{\sum_{j=0}^{n-1} M_j} \frac{e}{\Delta}; \quad (27)$$

When $(1+\Delta)p_I < \frac{1}{2}$, $\frac{(1+\Delta)p_I}{1-(1+\Delta)p_I} < 1$. Thus $\lim_{n \rightarrow \infty} \sum_{j=0}^{n-1} M_j < \infty$ for any $\{\varphi_j\}$ and $\phi \in (0, 1)$. That is to say, there exists a lower bound for z_0 , denoted it as \underline{z} . Since $\sum_{j=0}^{n-1} M_j = \frac{1}{z_0}$, the total discounted agency rent is a continuous function of z_0 . Because $[\underline{z}, 1]$ is a compact set, the total discounted agency rent as a continuous function must reach its minimum at some $z_0^* \in [\underline{z}, 1]$.

When $(1+\Delta)p_I \geq \frac{1}{2}$, $\lim_{n \rightarrow \infty} \sum_{j=0}^{n-1} M_j = \infty$. Since M_j is increasing, then for any finite T

$$\begin{aligned} & \lim_{n \rightarrow \infty} \frac{\sum_{j=0}^{n-1} M_j \sum_{k=0}^j \beta^k}{\sum_{j=0}^{n-1} M_j} \frac{e}{\Delta} + \frac{\frac{\beta}{1-\beta}(1 - (1+\Delta)((1-\varphi_0)p_C + \varphi_0 p_I))}{\sum_{j=0}^{n-1} M_j} \frac{e}{\Delta} \\ & \geq \lim_{n \rightarrow \infty} \frac{\sum_{j=0}^{n-1} M_j \sum_{k=0}^j \beta^k}{\sum_{j=0}^{n-1} M_j} \frac{e}{\Delta} \\ & > \sum_{k=0}^T \beta^k \frac{e}{\Delta} \\ & > \frac{1}{1-\beta} \frac{e}{\Delta}. \end{aligned}$$

From proof of Lemma 1, Result 3 we know that when $z_0 = 1$, the total agency rent is

$$\begin{aligned} & \frac{e}{\Delta} + \frac{\beta}{1-\beta} [\phi(1 - (1 + \Delta)p_I) + (1 - \phi)(1 - (1 + \Delta)p_C)] \frac{e}{\Delta} \\ & < (1 + \frac{\beta}{1-\beta}) \frac{e}{\Delta} \\ & = \frac{1}{1-\beta} \frac{e}{\Delta}. \end{aligned}$$

Thus there exists a finite T and corresponding \underline{z} such that any $z_0 \in (0, \underline{z})$ is strictly dominated by $z_0 = 1$. Then we only need to consider the compact set $[\underline{z}, 1]$, again the total discounted agency rent as a continuous function must reach its minimum at some $z_0^* \in [\underline{z}, 1]$. \square