Online Appendix: Extended Discussions
for ***Persuasion in Relationship Finance***

OA1. *Restricted information design space*

Thus far we have assumed that the contracting space and information design space coincide, that is, the entrepreneur is able to generate arbitrarily informative signals about the eventual cash flow. As is typical in the literature, this is justified by interpreting X as the most informative signal the entrepreneur can generate while conducting his usual entrepreneurial activities (Kamenica and Gentzkow, 2011).

We next show that our main results, IPH and the optimality of convertible securities, continue holding when we relax this assumption, as long as we allow binary experiments with a threshold scheme that generate a high signal for X sufficiently large and a low signal otherwise. Doing so also demonstrates that all our main results are robust to having a deterministic ε at $t = 0$.

Specifically, we depart from the baseline setup to assume that the final cash flow is $Y(X, \delta_1, \delta_2)$, where $Y(\cdot, \cdot, \cdot)$ is strictly increasing in X and δ_t, $t = 1, 2$, are random variables being realized and publicly observed at the beginning of period t. We denote their support by $[\underline{\delta}_t, \bar{\delta}_t]$, $t = 1, 2$. We further denote the probability density of Y by $f_Y(\cdot)$.

The security used to finance the project is $s(Y)$ when it is exogenous and $\{\lambda, s_I(Y), s_O(Y)\}$ when it is endogenously designed at $t = 0$. In other words, the contracting is over the cash flow Y while the information production and design is over a restricted space X. To be consistent with our baseline model, we further assume that $\mathbb{E}[\varepsilon + Y - I|\delta_1] < 0$ for all realizations of δ_1, that is, experimentation is necessary to justify the interim investment.

Note that this specification also allows the entrepreneur's private benefit $\varepsilon \in (0, \bar{\varepsilon})$ to be deterministic at $t = 0$, that is, $g(\varepsilon)$ can be an atom. This matters little for Proposition OA1, but allows us to demonstrate in Proposition OA2 that convertible securities are optimal even when ε is common knowledge at the time of relationship formation.

In Proposition OA1, we first verify the presence of IPH without long-term contracting.

Proposition OA1. *Consider the setting illustrated in Figure 1 with the modifications mentioned above. For any K and $s(\cdot)$, relationship financing is infeasible for sufficiently high and sufficiently low levels of interim competition (captured by $1 - \mu$).*

Proof. We modify the expressions for the expected payoffs, and similar to the baseline case, we show that the entrepreneur chooses a threshold scheme for disclosing X, for every realization of δ_1. Then, we verify the presence of IPH by showing that for low enough levels of interim competition (large enough μ), the threshold is chosen such that the insider just breaks even at the interim period, and hence, she cannot recoup the initial cost K.

Similar to (7), the entrepreneur designs an experiment that solves the following optimization problem, given δ_1:

$$\max_{(\mathcal{Z},\pi)} \mathbb{E}\left[\mathbb{E}\left[\varepsilon + Y - \mu s(Y) - (1-\mu)I|z, \delta_1\right] \mathbb{I}_{\{\mathbb{E}[s(Y)|z,\delta_1] \geq I\}}\right] \tag{OA.1}$$

The solution to (OA.1) is a binary experiment with a threshold scheme, since the entrepreneur's indirect payoff from the continuation at X, i.e. $\mathbb{E}[\varepsilon + Y - \mu s(Y) - (1-\mu)I|X, \delta_1]$, is weakly increasing in X. To see this, note that:

$$\mathbb{E}[\varepsilon + Y - \mu s(Y) - (1-\mu)I|X,\delta_1] = \mathbb{E}[\varepsilon + (1-\mu)(Y-I)|X,\delta_1] + \mathbb{E}[\mu(Y - s(Y))|X,\delta_1]$$

Both terms on the RHS are weakly increasing in Y and Y is strictly increasing in X. Therefore, the LHS is weakly increasing in X. When $\mu \in (0,1)$, the LHS is strictly increasing in X.

Furthermore, note that for sufficiently large values of μ, more specifically if $\varepsilon > (1-\mu)I$, the entrepreneur's payoff from continuation would be strictly positive for all values of Y, since both $Y - \mu s(Y)$ and $\varepsilon - (1-\mu)I$ are positive. Therefore, for such values of μ, the entrepreneur chooses the threshold $\bar{X}(\delta_1)$ such that $\mathbb{E}[s(Y)|X \geq \bar{X}(\delta_1),\delta_1] = I$, that is, the project is financed when $X \geq \bar{X}(\delta_1)$ and the insider merely breaks even from the continuation, which means she cannot cover the initial cost K for sufficiently low levels of interim competition. Here we note that the condition $\mathbb{E}[\varepsilon + Y - I|\delta_1] < 0$ implies that $\bar{X}(\delta_1) > 0$ for all realizations of δ_1. Hence the threshold is interior.

Next, the insider cannot recover K with sufficiently intense interim competition (for sufficiently low μ) because her information monopoly rent vanishes as μ goes to 0. The proposition ensues. □

Proposition OA1 shows that IPH does not require the entrepreneur's having access to arbitrarily informative experiments. In fact, as long as the entrepreneur's information input is marginal for the continuation decision, the insider investor would be held up by the entrepreneur's inefficient information production, especially when the level of interim competition is low.

Proposition OA2 shows that under general conditions, the optimal contract involves a security with a debt-like flat region to help the entrepreneur internalize the cost of inefficient continuation. The optimality of convertible securities therefore does not hinge on our specific baseline assumptions.

Proposition OA2. *Suppose $X^*(\delta_1)$ is the solution to $\mathbb{E}[Y|X = X^*(\delta_1);\delta_1] = I - \varepsilon$. Furthermore, define the probability density $\tilde{f}_{Y,\delta_1}(Y) \equiv f_Y(Y|X = X^*(\delta_1);\delta_1)$. If $\tilde{f}_{Y,\delta_1}(Y) \succ_{FOSD} \tilde{f}_{Y,\delta_1'}(Y)$ for $\delta_1 > \delta_1'$, then the optimal security for the insider financier, $s_I(Y)$, is flat in the region $[\underline{Y},\bar{Y}]$, where $\bar{Y} \equiv \sup\{Y|\tilde{f}_{Y,\delta_1}(Y) > 0\}$ and $\underline{Y} \equiv \inf\{Y|\tilde{f}_{Y,\delta_1}(Y) > 0\}$.*

Proof. To prove the proposition, we first find the entrepreneur's expected continuation payoff for every X, for a given contract $\{s_I(\cdot), s_O(\cdot), \lambda\}$. The M function introduced in (14) would be modified. Then, we show that to implement the optimal information design, which requires sending a high signal iff $X \geq X^*(\delta_1)$, $s_I(\cdot)$ needs to be flat in $[\underline{Y},\bar{Y}]$. Note that the expected social surplus for a given pair of (X,δ_1) is $\mathbb{E}[Y - I + \varepsilon|X,\delta_1]$. Since Y is strictly increasing in X and $\mathbb{E}[Y - I + \varepsilon|X = X^*(\delta_1),\delta_1] = 0$, by definition, the investment is socially optimal if and only if $X \geq X^*(\delta_1)$.

The entrepreneur's expected payoff from continuation at X, for a given δ_1, is:

$$\tilde{M}(X; s_I, s_O, \lambda, \delta_1) = \mathbb{E}[\varepsilon + Y - s_I(Y) - (1-\lambda)I|X,\delta_1]. \quad \text{(OA.2)}$$

A necessary condition for the threshold experiment $X \geq X^*(\delta_1)$ to be optimal for the entrepreneur is that $\tilde{M}(X) \geq 0$ if and only if $X \geq X^*(\delta_1)$, for all realizations of δ_1. Since $\tilde{M}(\cdot)$ is strictly increasing in X, this necessary condition can be translated to $\tilde{M}(X^*(\delta_1)) = 0$, for all δ_1, which implies:

$$\mathbb{E}[\varepsilon + Y - s_I(Y) - (1-\lambda)I|X = X^*(\delta_1),\delta_1] = 0 \Rightarrow \mathbb{E}[s_I(Y)|X = X^*(\delta_1),\delta_1] = \lambda I. \quad \text{(OA.3)}$$

Noting that $\tilde{f}_{Y,\delta_1}(Y) \equiv f_Y(Y|X = X^*(\delta_1), \delta_1)$, we can rewrite (OA.3) as

$$\int_{\underline{Y}}^{\bar{Y}} s_I(Y)\tilde{f}_{Y,\delta_1}(Y)dY = \lambda I, \tag{OA.4}$$

where $\tilde{f}_{Y,\delta_1}(\cdot)$ is strictly monotone in the sense of first-order stochastic dominance. Furthermore, note $s_I(Y)$ is weakly increasing in Y. Therefore, if $s_I(Y_1) > s_I(Y_2)$ for some $Y_1 > Y_2$, where $Y_1, Y_2 \in [\underline{Y}, \bar{Y}]$, then the integral in (OA.4) would be increasing in δ_1, which is a contradiction. Therefore, $s_I(Y)$ should be constant over $[\underline{Y}, \bar{Y}]$ in order to implement the socially optimal information design, and hence, the socially optimal investment decision.

□

To maximize his expected payoff when forming the financing relationship at $t = 0$, the entrepreneur would like to generate the continuation signal if and only if $X \geq X^*(\delta_1)$ because doing so generates the maximum social surplus which ex ante all accrue to him. He therefore needs a security that helps him internalize the cost of inefficient continuation during the interim regardless of the realization of δ_1. In other words, the optimal security has to be flat over the entire region of Y that is realizable conditional on any realization of δ_1 and $X = X^*(\delta_1)$.

OA2. *Renegotiation and contracts with exogenous securities*

This section examines the IPH in two alternative contracting arrangements. In the first part, we allow for a form of imperfect contracting, in which the insider renegotiates the security $s(\cdot)$ when she makes the continuation decision. In the second part, we allow for long-term contracts specifying the fraction of investments should be raised from the insider and outsiders for the continuation, while the form of securities is determined exogenously.

Renegotiation. The setup with renegotiation follows the baseline model in Section 3, with the only difference being that the insider can renegotiate on security $s(\cdot)$, following the signal realization of the entrepreneur's experiment. For simplicity, let $\mu = 1$, implying the project should be financed in its entirety by the insider, and the insider has full bargaining power in the renegotiation process. Given this, the insider clearly demands all the project's cash flow upon continuation. Therefore, the only renegotiation-proof security is $s(X) = X$. According to Corollary 1, the entrepreneur chooses an experiment so that the insider cannot recoup her initial cost, rendering the financing relationship impossible, despite the insider having the whole bargaining power over the choice of security. This result indicates the IPH is robust to renegotiation.

Long-term contracts with exogenous securities. The setup follows Section 4 except that we restrict our attention to long-term contracting with exogenous security types in the next lemma. For example, if the security type is exogenously restricted to debt contracts, then $s(\cdot)$ is simply pinned down by the face value; if it is restricted to equity contracts, then $s(\cdot)$ is specified by the number of shares. These restricted contracts partially resolve the traditional information hold-up problem (e.g., Von Thadden, 1995), but not IPH. Security design is therefore integral to resolving IPH.

Lemma OA1 (Contracting with Exogenous Security Design).

(a) For any given security $s(\cdot)$, without outsider investors ($\lambda = 1$, $s_I(X) = s(X)$), the equilibrium is unique and no project is financed at $t = 0$ unless $K = 0$.

(b) For any given security $s(\cdot)$, $\exists \lambda \in (0, 1)$ such that the insider receives positive interim rent under the contract $\{\lambda s(\cdot), (1-\lambda)s(\cdot), \lambda\}$.

(c) No contract with a single form of security (including debt, equity, or call options) for both the insider and outsiders implements the first-best social outcome.

Proof. **Part (a).** When $\lambda = 1$, either the insider invests, or she does not and nor do the outsiders after observing the insider's action. Therefore, the insider can fully squeeze the entrepreneur, which is equivalent to the case of $\mu = 1$ in Proposition 1. As is shown in Corollary 1, the relationship financing is infeasible in this case.

Part (b). We first show that λ enters into the entrepreneur's payoff in a way similar to that of μ in (5). We then show the insider gets a positive expected interim payoff (after investing K) for values of $\lambda > 0$ such that $\hat{X}(\lambda) > \bar{X}$.

Note following signal $z \in \mathcal{Z}$, the insider chooses to invest by paying $p^I = \lambda I$ if and only if $\lambda \mathbb{E}[s(X) - I|z] \geq 0$. Following the insider's action, the outsiders learn if the project has a positive conditional expected payoff, that is, $z \in \mathcal{Z}^+$. Therefore, following the insider's investment, the outsiders pay the entrepreneur $p^O = (1-\lambda)\mathbb{E}[s(X)|z \in \mathcal{Z}^+]$. Therefore, the entrepreneur's payoff from experiment $(\mathcal{Z}, \pi)$ is given by:

$$
\begin{aligned}
U^E(\mathcal{Z}, \pi; \lambda, \varepsilon) &= \sum_{z \in \mathcal{Z}^+} \int_0^1 (\varepsilon + X - I - s(X) + p^I + p^O)\pi(z|X)f(X)dX \\
&= \sum_{z \in \mathcal{Z}^+} \int_0^1 (\varepsilon + X - I - s(X) + \lambda I + \mathbb{E}[s(X)|z \in \mathcal{Z}^+])\pi(z|X)f(X)dX \\
&= \int_0^1 \sum_{z \in \mathcal{Z}^+} (\varepsilon + X - I)\pi(z|X)f(X)dX - \lambda \int_0^1 \sum_{z \in \mathcal{Z}^+} (s(X) - I)\pi(z|X)f(X)dX
\end{aligned}
$$

Note the entrepreneur's preference over different experiments exactly coincides with (5), had we replaced μ with λ. Similarly, the insider's expected interim payoff is:

$$
U^I(\mathcal{Z}, \pi; \lambda) = \lambda \mathbb{E}[(s(X) - I)\mathbb{I}_{\{X \geq \max\{\bar{X}, \hat{X}(\lambda)\}\}}] \tag{OA.5}
$$

By definition, $\mathbb{E}[(s(X) - I)\mathbb{I}_{\{X \geq \bar{X}\}}] = 0$ and $\hat{X}(0) = I - \varepsilon > \bar{X}$. Hence, for $\lambda > 0$ sufficiently close to zero, $\hat{X}(\lambda) > 0$ and consequently (OA.5) is positive, which completes the proof.

Part (c). The investment is socially efficient if and only if $X \geq I - \varepsilon$. However, for contracts of the form $\{\lambda s(\cdot), (1-\lambda)s(\cdot), \lambda\}$, investment takes place if $X \geq \max\{\bar{X}, \hat{X}(\lambda)\}$, as shown in the previous part. Since both $\bar{X}$ and $\hat{X}(\lambda)$ are less than $I - \varepsilon$ for $\lambda > 0$, the investment is inefficient with a positive probability (overinvestment for $X \in [\max\{\bar{X}, \hat{X}(\lambda)\}, I - \varepsilon)$) for $\lambda > 0$. Moreover, relationship financing is impossible for $\lambda = 0$, when $K > 0$. It completes the proof. □

Recall that contract $\{\lambda s(\cdot), (1-\lambda)s(\cdot), \lambda\}$ allows the insider financier to purchase λ fraction of the securities issued at $t = 1$, and sells the remainder to competitive arms-length investors. Part (a) extends Proposition 1 and Corollary 1 to the case where the entrepreneur commits to a long-term contract and consequently the insider does not have full bargaining power from making TIOLI offers during the interim. IPH is thus robust to whether the insider has an information monopoly. More generally, one can prove that when the contracts can be renegotiated to leave the insider an intermediate level of bargaining power, IPH persists, just like our baseline case without long-term contracts.

Part (b) reveals that the insider's interim rent becomes positive for some $\lambda < 1$. This reflects the importance of "second-sourcing" the investment I (e.g., Farrell and Gallini (1988)), i.e., committing to $\lambda < 1$, because the interim rent is decreasing in λ when λ is big. Even the insider investor has the incentive to decrease his shares of the surplus somewhat to enable the entrepreneur to internalize the cost of inefficient continuation, and thus creating a larger social surplus. Note that although λ here has a similar effect as μ, it is a contract design parameter rather than an exogenous friction.

Finally, Part (c) reveals that long-term contracting with a single form of security cannot restore social efficiency. Given that the entrepreneur gets all the expected social surplus at $t = 0$, her ex ante payoff is not maximized either.

OA3. *Optimal design of a single security*

Due to regulatory concerns, issuance costs, or high cost of communicating with outsiders in the later rounds, the entrepreneur often can only issue one single form of security to both insiders and outsiders. For example, in venture financing the right of first refusal gives its holder the contractual rights but not the obligation to purchase new security issuance before others can purchase that *same security with the same terms*; banks by regulation can only use debt contracts.

As follows, we show that if the entrepreneur can commit to financing λI from the outsider, the unique optimal security is equity. In the absence of such commitment, the optimal security is indeterminate. However, the set of optimal securities include debt contracts, but not necessarily equity contracts. For notational simplicity, we write ε as if it is deterministic. What is key is that some interim information (ε included) affecting the entrepreneur's information production is non-contractible at $t = 0$.

Specifically, the entrepreneur issues security contract $s(X)$ and determines the fraction to the insiders λ, i.e., $s_I(X) = \lambda s(X)$ and $s_O(X) = (1 - \lambda)s(X)$. Proposition OA3 characterizes the equilibrium security, optimal level of commitment to outsider competition, and the optimal information production.

Proposition OA3 (Optimality of Equity)**.** *Under the single-security constraint and for a given insider's experiment $(\mathcal{Y}, \omega_q)$, the entrepreneur optimally issues equities to both the insider and the outsiders. In particular, $s(X) = X$, hence $s_I(X) = \lambda^* X$ and $s_O(X) = (1 - \lambda^*)X$ for some $\lambda^* \in (0, 1)$. The optimal security is unique and it does not implement the first-best outcome if $K > 0$.*

Proof. We prove the proposition for the case of an unsophisticated insider, where she has no proprietary information about the outcome, other than what is provided by the entrepreneur's experiment. The proof for the case of sophisticated investors is similar. The single-security constraint implies that the entrepreneur chooses from contracts in the form of $\{\lambda s(X), (1 - \lambda)s(X), \lambda\}$. Therefore, every contract can be represented by the pair of $\{s(X), \lambda\}$.

Denote the entrepreneur's indirect utility from investment at X by $M^s(s, \lambda)$, where:

$$M^s(X; s, \lambda) = \varepsilon + X - \lambda s(X) - (1 - \lambda)I = M(X; \lambda s(.), (1 - \lambda)s(.), \lambda) \quad \text{(OA.6)}$$

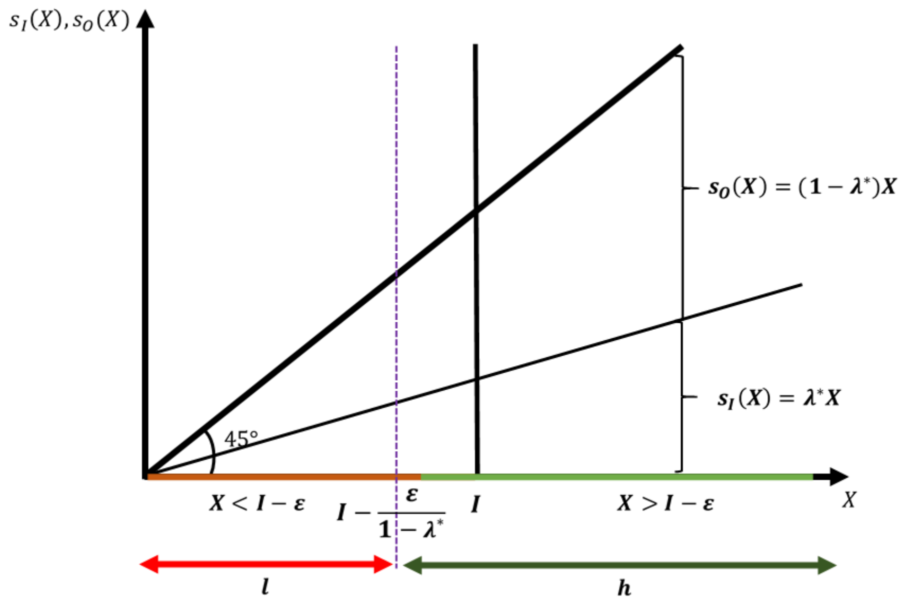

Figure OA1: The optimal securities for the entrepreneur under one-security condition; specific example of without investor sophistication.

The entrepreneur then solves the following maximization problem:

$$\begin{aligned} &\max_{s(.),\lambda,X_c} \mathbb{E}[M^s(X;s,\lambda)\mathbb{I}_{\{M^s(X;s,\lambda)\geq 0\}}] \\ &s.t. \quad \lambda\mathbb{E}[(s(X)-I)\mathbb{I}_{\{M^s(X;s,\lambda)\geq 0\}}] \geq K \end{aligned} \tag{OA.7}$$

Note that the objective function in (OA.7) is bounded by $\mathbb{E}[\max\{\varepsilon + X - I, 0\}]$ and the constraint constitutes a closed and bounded subset in a $\mathcal{L}^1 \times [0,1]$ space that contains all combination of regular securities and λ. Therefore, the optimal contract exists. We denote the security $s(X) = X$, by $s_i(\cdot)$.

Definition OA1. *Consider the contract* $\{s(.),\lambda\}$*. If* $M^s(0;s,\lambda) < 0$*, then* $\hat{X}(s(.),\lambda)$ *is defined to be the solution to* $M^s(\hat{X}(s(.),\lambda);s,\lambda) = 0$*; otherwise, we define* $\hat{X}(s(.),\lambda) = 0$.

The next lemma follows immediately from the expression (OA.6) and Definition OA1.

Lemma OA2. *Suppose* $s_1(.)$ *and* $s_2(.)$ *are two regular securities such that* $s_1(X) \geq s_2(X)\ \forall\ X \in [0,1]$.
(a) $\hat{X}(s_1,\lambda)$ *is weakly decreasing in* λ.
(b) $\hat{X}(s_1,\lambda) \geq \hat{X}(s_2,\lambda)$*, for every* $\lambda \in [0,1]$.

Now, we are ready to prove the optimality of equity. Consider a contract $\{s_1,\lambda_1\}$ that satisfies the constraint in (OA.7). Part (b) in Lemma OA2 implies:

$$\lambda_1\mathbb{E}[(s_1(X)-I)\mathbb{I}_{\{X\geq\hat{X}(s_1(.),\lambda_1)\}}] \leq \lambda_1\mathbb{E}[(X-I)\mathbb{I}_{\{X\geq\hat{X}(s_i(.),\lambda_1)\}}] \tag{OA.8}$$

Then there exists $\lambda_i \leq \lambda_1$ such that $\lambda_i \mathbb{E}[(X-I)\mathbb{I}_{\{X \geq \hat{X}(s_i(.),\lambda_i)\}}] = K$. Next, we show that the entrepreneur prefers the contract $\{s_i(.), \lambda_i\}$ to $\{s_1, \lambda_1\}$:

$$\mathbb{E}[M(X; s_i, \lambda_i)\mathbb{I}_{\{X \geq \hat{X}(s_i(.),\lambda_i)\}}] = \mathbb{E}[(\varepsilon + (1-\lambda_i)(X-I))\mathbb{I}_{\{X \geq \hat{X}(s_i,\lambda_i)\}}] = \mathbb{E}[(\varepsilon + X - I)\mathbb{I}_{\{X \geq \hat{X}(s_i,\lambda_i)\}}] - K$$
$$\geq \mathbb{E}[(\varepsilon + X - I)\mathbb{I}_{\{X \geq \hat{X}(s_1(.),\mu_1)\}}] - \lambda_1 \mathbb{E}[(s_1(X) - I)\mathbb{I}_{\{X \geq \hat{X}(s_1(.),\lambda_1)\}}] = \mathbb{E}[M(X; s_1, \lambda_1)\mathbb{I}_{\{X \geq \hat{X}(s_1(.),\lambda_1)\}}]$$

Therefore, the optimal contract under the single security constraint is equity. For the case with sophisticated investors, λ^* depends on the insider's experiment, although an equity is still uniquely optimal.

□

Proposition OA3 shows that the optimal single security is equity when commitment in λ is feasible. Recall that the entrepreneur uses the security to best commit himself to efficient interim information production. With a single security, the entrepreneur cannot fully allocate the downside exposure to himself because both the insider and outsiders get the same security, and the outsiders are just a pass-through of interim surplus to the entrepreneur. However, by reducing λ, we naturally reduce the insider's exposure relative to the entrepreneur. Equity is thus optimal when λ is endogenous because it gives the insider the largest upside, allowing her to get enough rent to cover K with the smallest λ. Figure OA1 displays the optimal security for the entrepreneur.

Proposition OA4 (Optimality of Debt Contracts)**.** *Under the single security-constraint and when committing to raising λI from the insider is not feasible, the optimal security is indeterminate but the set of optimal securities includes debt.*

Proof. When the entrepreneur cannot commit to the way he finances I, the insider gets positive rent only if the signal is private. Therefore, she demands all issues when the signal is good. To give the insider lower rent, the entrepreneur uses securities that make the insider break-even, i.e.

$$U^I(s(\cdot)) = \mu \mathbb{E}\left[(s(X) - I)\mathbb{I}_{\{X \geq \hat{X}(\mu; s(\cdot))\}}\right] = K \qquad \text{(OA.9)}$$

The entrepreneur's expected payoff for a given choice of $s(\cdot)$ from this set of securities is

$$U^E(s(\cdot)) = \mathbb{E}\left[(\varepsilon + X - \mu s(X) - (1-\mu)I)\mathbb{I}_{X \geq \hat{X}(\mu; s(\cdot))}\right] = \mathbb{E}\left[(\varepsilon + X - I)\mathbb{I}_{\{X \geq \hat{X}(\mu; s(\cdot))\}}\right] - K. \qquad \text{(OA.10)}$$

To maximize (OA.10), the entrepreneur chooses a security that maximizes $\hat{X}(\mu; s(\cdot))$. Note that $\hat{X}(\mu; s(\cdot))$ is the solution to the following equality:

$$\hat{X}(\mu; s(\cdot)): \quad \varepsilon + \hat{X}(\mu) - \mu s(\hat{X}(\mu)) - (1-\mu)I = 0 \Rightarrow \hat{X}(\mu) \leq I - \frac{\varepsilon}{1-\mu}$$

and equality holds if and only if $s(X) = X$ for $X \leq [I - \frac{\varepsilon}{1-\mu}]$. Therefore, all double-monotone securities with $\mathbb{E}\left[(s(X) - I)\mathbb{I}_{\{X \geq I - \frac{\varepsilon}{1-\mu}\}}\right] = K$ and $s(X) = X$ for $X \leq I - \frac{\varepsilon}{1-\mu}$ implement the optimal design for the entrepreneur. Clearly, a debt or a convertible security can implement these two conditions, but equity or call options cannot.

□

In Proposition OA4, we study the case that the entrepreneur can only commit to the single security he issues in the future. For example, the entrepreneur might commit to the form of security by choosing his institutional investor (Banks, Private Equities, Venture Capitalist, etc). In this case, if the commitment to the future round of investments is not feasible, then debt contracts become optimal. The intuition is the following: The security should be the most sensitive to the losses to discipline the entrepreneur the most. Since a debt contract is the most sensitive security in the down-side, it always can implement the optimal security and it is regardless of μ and the insider's experiment.

OA4. *Contracting under IPH*

The "efficient effort" in our case is to produce continuation only when $X + \varepsilon > I$, the cost of that effort is the loss of private benefit when we terminate projects. The "action" in our setup is the experimentation, and the outcomes of the experiments correspond to noisy signals of the action.

It is a well-known result that for risk-neutral agents, the optimal security is debt (Innes (1990)). Moreover, even when we can contract on noisy signals of an agent's action, the outcome is generally not first best (e.g., Holmstrom (1979)). Yet the optimal design is partially indeterminate in our model and restores social efficiency. This result only relies on the contractibility of actions the interim information leads to, namely continuation or termination. Our findings point to the key difference between IPH and those in conventional models.

This contrast derives from two subtle differences between our setting and the ones in Holmstrom (1979) and Innes (1990). First, the principal takes an action based on the information produced by the agent, which implies that the agent's effort can affect his final payoff through affecting the principal's continuation decision. In fact, we show in Section OA2 that by designing a security that makes the principal's continuation decision and thus the agent's payoff more sensitive to the agent's effort, we can also align the agent's incentives. Second, which is more important, the agent's effort in our model affects information production, but not the final output X, thus allowing the principle to know exactly how the entrepreneur's action has affected the final payoff. In other words, upon seeing the cash flow X, everyone knows if the continuation decision is socially optimal or not, hence the entrepreneur ex ante can design the security and contract contingent on the continuation payoff to perfectly incentivize the entrepreneur during the interim to take the right action. In some sense, the noise in the entrepreneur's action — signal to continue or terminate – is orthogonal to the continuation payoff, and making the agent's payoff contingent on both fully solves the agency problem. It is also worth pointing out that unlike the literature on costly experimentation and learning (e.g., Bergemann and Hege (1998) and Hörner and Samuelson (2013)) with hidden effort and hidden information, the principal in our setting observes perfectly the signal the agent produces, which is important for attaining the first-best outcome with the optimal contract.

OA5. *Insider's independent experimentation*

We revisit the entrepreneur's optimal information design, the entrepreneur and the insiders' equilibrium expected payoff and our contractual solution to IPH for the case that the signals of the entrepreneur's experiment are restricted to be independent of the insider's experiment, conditional on the outcome X. In Proposition OA5, we show that the U-shape pattern in the insider's payoff as a function of μ still persists. In Corollary OA1, consistent with Proposition 5 in Kolotilin (2018), we show the insider's payoff is not increasing in the informativeness of her signal. In Proposition OA6, we examine the effect of interim competition on the efficiency of investment decisions and the insider's interim payoff for $q \in \left[\frac{1}{2}, 1\right]$. For notational simplicity, we write ε as if it is deterministic and remind the reader that our results only rely on the non-contractibility at $t = 0$ of some interim information (ε included) affecting the entrepreneur's information production.

The results are provided for the specific case that the information structure of the insider's signal follows the one introduced in Example 1, with the only difference that q represents the probability of receiving signal $\tilde{h}_q$ ($\tilde{l}_q$) when $X - I \geq 0$ ($X - I < 0$). As a reminder, we repeat the structure of the insider's private information:

Assumption OA1. *The insider's experiment generates a binary signal, i.e.* $\mathcal{Y} = \{\tilde{h}, \tilde{l}\}$*, of the following information structure:*

$$\omega_q(\tilde{h}|X) = \begin{cases} q & I \leq X \leq 1 \\ 1 - q & 0 \leq X < I. \end{cases}$$

Proposition OA5 (General IPH)**.** *Suppose* $\bar{X}(q)$ *is the solution to the following equation:*

$$q \int_0^I (s(X) - I) f(X) dX + (1 - q) \int_I^1 (s(X) - I) f(X) dX = 0 \tag{OA.11}$$

Moreover, assume $q \geq \bar{q}$*, where* $\bar{q}$ *is the solution to equation* $\bar{q}^2 + \bar{q} = 1$*. (a) The entrepreneur chooses an experiment that sends a high signal if* $X \geq \max\{\bar{X}(q), \hat{X}(\mu)\}$ *and sends a low signal, otherwise.*

(b) Denote $\bar{K}_C^{\varepsilon}(\mu, q) \equiv \mu\mathbb{E}[(s(X) - I)\mathbb{I}_{X \geq \max\{\bar{X}(q), \hat{X}(\mu)\}}]$ *as the capacity of relationship formation for given* q *and* μ*. Then, there exists level of sophistication* q_1 *such that for every* $q \in (\frac{1}{2}, q_1)$*,* $\bar{K}_C^{\varepsilon}(\mu, q)$ *is U-shaped in* $[\hat{\mu}(q), 1]$ *for some* $\hat{\mu}(q) \in (0, 1)$*,.* $\bar{K}_C^{\varepsilon}(\mu, q)$ *is decreasing in* μ *for* $q \in [q_1, \bar{q}]$*.*

(c) As $\varepsilon \to 0$*, function* $\bar{K}_C^{\varepsilon}(\mu; q)$ *converges to* $\bar{K}_C^{0}(\mu; q)$*, which is increasing in* μ*.*

Proof. Similar to the proof of Lemma 1, we prove the lemma for regular securities $s(X)$. First we note that if $\mathbb{E}[s(X) - I|X > I] \leq 0$, then $\bar{X} \geq I$, where $\bar{X}$ is the threshold introduced in Proposition 1. In this case, the investor's signal y is not used and the equilibrium experiments, investment functions and payoffs are the same as the ones provided in Proposition 1. As such, the remaining proof is centered on the case $\mathbb{E}[s(X) - I|X > I] > 0$ for simplicity in exposition.

Proof for Part (a)

Similar to the proof of Proposition 1, first we show that for every experiment, there is another experiment with a bounded number of signals that gives the entrepreneur the same expected payoff. It helps us prove the existence of optimal experiments and then characterize them. We refer to the private information (broadly defined) the insider has as *investor type*.

Lemma OA3. *Denote* T *as the set of investor type and* A *the set of actions. As long as* A *and* T *are finite, for every experiment* $(\mathcal{Z}, \pi)$*, there exists experiment* $(\mathcal{Z}', \pi')$ *such that* $|\mathcal{Z}'| \leq |A|^{|T|}$ *and* $U^E(\mathcal{Z}', \pi') = U^E(\mathcal{Z}, \pi)$*.*

Proof. For every pure strategy of the investor, such as $\mathfrak{a}(\cdot) : T \to A$, define $\mathcal{Z}(\mathfrak{a})$ as the set of signals in $\mathcal{Z}$, such as z, that the investor chooses $\mathfrak{a}(t)$ when her type is t and she receives z. Note that if $\mathcal{Z}(\mathfrak{a})$ is non-empty, then:

$$\begin{aligned}&\mathbb{E}[u^I(\mathfrak{a}(t),X)|t,z] \geq \mathbb{E}[u^I(a',X)|t,z] \quad \forall t\in T, z\in\mathcal{Z}(\mathfrak{a})\\ \Rightarrow&\mathbb{E}[u^I(\mathfrak{a}(t),X)|t,\mathcal{Z}(\mathfrak{a})] \geq \mathbb{E}[u^I(a',X)|t,\mathcal{Z}(\mathfrak{a})] \quad \forall t\in T\end{aligned}$$

where $u^I(a,X)$ is the final payoff of the investor from action a in state X. Now define the experiment $(\mathcal{Z}',\pi')$ as follows: $\mathcal{Z}'=\{z_{\mathfrak{a}}\}_{\mathfrak{a}\in A^T}$, for all $\mathfrak{a}$ that $\mathcal{Z}(\mathfrak{a})$ is non-empty. Moreover, define

$$\pi'(z_{\mathfrak{a}}|X)=\sum_{z\in\mathcal{Z}(\mathfrak{a})}\pi(z|X) \quad \text{if } \mathcal{Z}(\mathfrak{a},t) \text{ is non-empty}$$

Note that by definition, $z_{\mathfrak{a}}$ is the signal in $\mathcal{Z}'$ that induces the strategy profile $\mathfrak{a}(t)$. We only need to show that $U^E(\mathcal{Z}',\pi')=U^E(\mathcal{Z},\pi)$. To see this,

$$\begin{aligned}U^E(\mathcal{Z}',\pi')&=\int_0^1\sum_{t\in T}\sum_{z_{\mathfrak{a}}\in\mathcal{Z}'}u^E(\mathfrak{a}(t),X)\pi'(z_{\mathfrak{a}}|X)g(t|X)f(X)dX\\ &=\int_0^1\sum_{t\in T}\left[\int_{z\in\mathcal{Z}}u^E(a(z,t),X)\pi(z|X)dz\right]g(t|X)f(X)dX=U^E(\mathcal{Z},\pi)\end{aligned}$$

where $a(z,t)$ is the investor's action for type t upon receiving signal z. □

Building from the above result, the next lemma proves the existence of an optimal experiment and characterizes it.

Lemma OA4. ***(a)*** *Suppose the investor's action is binary ($A=\{0,1\}$) and investor types are ordered by $T=\{t_1,t_2,\dots,t_{|T|}\}$ such that posteriors $u^I(1,X)-u^I(0,X)|t_i$ are ranked by first-order stochastic dominance. Then for every experiment, there is an experiment that implements the same expected payoffs and uses at most $|T|+1$ signals.* ***(b)*** *Under these conditions, an optimal experiment exists.*

Proof.

Proof of Part (a)

Note the single-crossing condition implies $a(z,t)$ to be weakly increasing in t. Therefore, there are at most $|T|+1$ functions of the form $\mathfrak{a}:T\to A$ that $\mathcal{Z}(\mathfrak{a})$ is non-empty. With an argument similar to that of Lemma A1(a) , one can construct an experiment with at most $|T|+1$ signals that implement the same expected payoffs.

Proof of Part (b)

To show the existence of the optimal experiment, we only need to look at the experiments with at most $|T|+1$ signals. In particular, we only need to show that the conditional distributions $\pi(z|X)$, $z\in\mathcal{Z}$, constitute a closed bounded set in $\mathcal{L}^{1|T|+1}$.

For an experiment $(\mathcal{Z},\pi)$, we can assume it has at most one signal $z_i\in\mathcal{Z}$ such that the entrepreneur chooses $a=1$ only if her type is t_i or above. $z_{|T|+1}$ is the signal following which no type would invest. Therefore, the part (a) shows that every experiment implements the same expected payoffs as the following experiment:

$$\int_0^1 (u^I(1,X) - u^I(0,X))\pi(z_i|X)g(t_j|X)f(X) \geq 0 \quad \text{iff } j \geq i, \; \forall 1 \leq i \leq |T|+1$$
$$\sum_{i=1}^{|T|+1} \pi(z_i|X) = 1 \quad \forall X \in [0,1], \quad \textit{and} \quad \pi(z_i|X) \geq 0 \quad \forall X \in [0,1], z_i \in \mathcal{Z} \tag{OA.12}$$

It is easy to check that the set of experiments satisfying (OA.12) is closed and bounded. Therefore, an optimal experiment exists.

□

As a result of the lemma and the corollary, the optimal experiment exists and has at most three signals. An exhaustive list of candidate signal ranges for such an optimal experiment is $\{m,l\}$, $\{h,l\}$ or $\{m,h,l\}$, where the investor only invests if she receives $(m,\tilde{h})$, $(h,\tilde{l})$ or $(h,\tilde{h})$. We show that for $q \leq \bar{q}$, the optimal experiment is essentially unique and it is of the second form.

Lemma OA5. *No two-signal experiment in the form of* $(\{l,m\},\pi)$*, where the investor invests iff she receives* $(m,\tilde{h})$*, is optimal.*

Proof. Suppose the contrary and suppose that there exists an optimal experiment $(\{m,l\},\pi_M^*)$. Therefore, π_M^* solves the following maximization problem:

$$\max_{\pi(m|X)} (1-q)\int_0^I (\varepsilon + X - s(X))\pi(m|X)f(X)dX + q\int_I^1 (\varepsilon + X - s(X))\pi(m|X)f(X)dX$$
$$s.t. \quad (1-q)\int_0^I (s(X)-I)\pi(m|X)f(X)dX + q\int_I^1 (s(X)-I)\pi(m|X)f(X)dX \geq 0$$
$$\pi(m|X) \in [0,1] \qquad \forall X \in [0,1]$$

Let κ be the multiplier corresponding to the constraint. π_M^* then maximizes the following objective function, given the constraint $\pi_M^*(m|.) \in [0,1]$.

$$\max_{\pi(m|X)} (1-q)\int_0^I [\varepsilon + X - s(X) + \kappa(s(X)-I)]\pi(m|X)f(X)dX$$
$$+q\int_I^1 [\varepsilon + X - s(X) + \kappa(s(X)-I)]\pi(m|X)f(X)dX$$

Since both $X - s(X)$ and $s(X) - I$ are weakly increasing functions, there is a threshold value $X_m \in [0,1]$ such that for all $X \geq X_m$, the expression in the bracket is non-negative. According to Lemma A1, the optimal experiment among those that only implement signals m and l has a threshold scheme, where the threshold X_m satisfies the following:

$$(1-q)\int_{X_m}^I (s(X)-I)f(X)dX + q\int_I^1 (s(X)-I)f(X)dX = 0$$

In this case, the expected utility of the entrepreneur from $(\{m,l\},\pi_M^*)$ is:

$$U^E(\{m,l\},\pi_M^*) = (1-q)\int_{X_m}^{I}(\varepsilon + X - s(X))f(X)dX + q\int_{I}^{1}(\varepsilon + X - s(X))f(X)dX \tag{OA.13}$$

By comparing the recent equality with (OA.11), it is easy to see that $X_m < \bar{X}(q)$. Now, we show how the entrepreneur can improve upon π_M^* by introducing signal h (a signal that induces investment regardless of the signal the investor receives). To show this, we consider two cases:

- $s(I) < I$: In this case, we can find a subset $A \subset [I,1]$ such that $\int_A (s(X) - I)f(X)dX = 0$. Then an experiment that sends h for the members of A ($\pi(h|X) = 1$ iff $X \in A$) and sends m for $[X_m, 1] \setminus A$ implements higher payoff for the entrepreneur by $(1-q)\int_A (\varepsilon + X - s(X))f(X)dX$.
- $s(I) = I$: Since $X - s(X)$ is a weakly increasing function, it implies that $s(X) = X$ for all $X \leq I$. Consider small positive values $\eta_1, \eta_2, \eta_3 \geq 0$ that satisfy the following:

$$(1-q)\int_{X_m+\eta_1}^{I-\eta_2}(s(X)-I)f(X)dX + q\int_{I+\eta_3}^{1}(s(X)-I)f(X)dX \geq 0$$

$$q\int_{I-\eta_2}^{I}(s(X)-I)f(X)dX + (1-q)\int_{I+\eta_3}^{1}(s(X)-I)f(X)dX \geq 0$$

and introduce the following alternative experiment $(\{l,m,h\},\tilde{\pi}_M)$:

$$\tilde{\pi}_M(h|X) = \begin{cases} 1 & X \in [I-\eta_2, I+\eta_3) \\ 0 & X \in [0, I-\eta_2) \cup [I+\eta_3, 1] \end{cases} \qquad \tilde{\pi}_M(m|X) = \begin{cases} 1 & X \in [X_m+\eta_1, I-\eta_2) \cup [I+\eta_3, 1] \\ 0 & X \in [0, X_m+\eta_1) \cup [I-\eta_2, I+\eta_3) \end{cases}$$

It is easy to verify that the experiment $\tilde{\pi}_M$ is designed in a way that the investor invests iff she receives one of $(m,\tilde{l})$, $(h,\tilde{l})$ or $(h,\tilde{h})$. Now, the difference in expected payoffs for the entrepreneur is given by:

$$U^E(\{m,l\},\tilde{\pi}_M) - U^E(\{m,l\},\pi_M^*) = -(1-q)\int_{X_m}^{X_m+\eta_1}(\varepsilon + X - s(X))f(X)dX$$
$$+ q\int_{I-\eta_2}^{I}(\varepsilon + X - s(X))f(X)dX + (1-q)\int_{I}^{I+\eta_3}(\varepsilon + X - s(X))f(X)dX$$

Now consider the contrary, that the introduced experiment is an optimal experiment. Then $\eta_1^* = \eta_2^* = \eta_3^* = 0$ should satisfy the first-order conditions for the following maximization problem:

$$\max_{\eta_1,\eta_2,\eta_3} \quad -(1-q)\int_{X_m}^{X_m+\eta_1}(\varepsilon + X - s(X))f(X)dX + q\int_{I-\eta_2}^{I}(\varepsilon + X - s(X))f(X)dX$$
$$+(1-q)\int_{I}^{I+\eta_3}(\varepsilon + X - s(X))f(X)dX$$

$$s.t. \quad \eta_1, \eta_2 \geq 0, \qquad q \int_{I-\eta_2}^{I} (s(X) - I) f(X) dX + (1-q) \int_{I}^{I+\eta_3} (s(X) - I) f(X) dX \geq 0, \quad and$$

$$(1-q) \left[\int_{X_m}^{X_m + \eta_1} (s(X) - I) f(X) dX + \int_{I-\eta_2}^{I} (s(X) - I) f(X) dX \right] + q \int_{I}^{I+\eta_3} (s(X) - I) f(X) dX \leq 0$$

Let κ_1 and κ_2 be the multipliers for the first constraints, respectively. Since $s(I) = I$, the FOC for η_2 is positive at $\eta_2 = 0$: $[\eta_2]|_{\eta_2 = 0} : q\varepsilon f(I)$.

Similarly, the FOC for η_3 is positive at $\eta_3 = 0$. Therefore, the optimal values are non-zero. It shows that the experiment $(\{m, l\}, \pi^*_M)$ cannot be optimal for any $q \in (\frac{1}{2}, 1)$.

□

Lemma OA6. *No three-signal experiment in the form of* $(\{l, m, h\}, \pi)$*, whereby signals* m *and* h *are both sent with positive probability, is optimal.*

Proof. First note we are considering the parameter range $q \in (\frac{1}{2}, \bar{q}]$. Suppose the contrary and there exists a three-signal optimal experiment $(\{l, m, h\}, \pi^*_{HM})$, where the investor invests iff she receives one of $(m, \tilde{l})$, $(h, \tilde{l})$ and $(h, \tilde{h})$. Then $\pi^*_{HM}(m|X)$ and $\pi^*_{HM}(h|X)$ solve the following optimization problem.

$$\begin{aligned} \max_{\pi(h|X), \pi(m|X)} & \int_0^I (\varepsilon + X - s(X))(\pi(h|X) + (1-q)\pi(m|X)) f(X) dX \\ & + \int_I^1 (\varepsilon + X - s(X))(\pi(h|X) + q\pi(m|X)) f(X) dX \end{aligned} \tag{OA.14}$$

$$\begin{aligned} s.t. \quad & q \int_0^I (s(X) - I)\pi(h|X) f(X) dX + (1-q) \int_I^1 (s(X) - I)\pi(h|X) f(X) dX \geq 0 \\ & (1-q) \int_0^I (s(X) - I)\pi(m|X) f(X) dX + q \int_I^1 (s(X) - I)\pi(m|X) f(X) dX \geq 0 \\ & \pi(h|X), \pi(m|X) \in [0, 1] \end{aligned}$$

Let λ^h and λ^m be the multipliers for the first two restrictions, respectively. Define $c_m(X)$ and $c_h(X)$ as follows:

$$c_h(X) = \begin{cases} \varepsilon + X - s(X) + q\lambda^h(s(X) - I) & 0 \leq X < I \\ \varepsilon + X - s(X) + (1-q)\lambda^h(s(X) - I) & I \leq X \leq 1 \end{cases}$$

$$c_m(X) = \begin{cases} (1-q)(\varepsilon + X - s(X) + \lambda^m(s(X) - I)) & 0 \leq X < I \\ q(\varepsilon + X - s(X) + \lambda^m(s(X) - I)) & I \leq X \leq 1 \end{cases}$$

Then $\pi(h|X)$ and $\pi(m|X)$ solves the following optimization problem subject to $0 \leq \pi(h|X), \pi(m|X) \leq 1$

$$\max_{\pi(h|X), \pi(m|X) \in [0,1]} \int_0^1 [c_h(X)\pi(h|X) + c_m(X)\pi(m|X)] f(X) dX \tag{OA.15}$$

Now, we appeal to Lemma A1. Note that the optimization problem (OA.15) is linear in $\pi(h|X)$ and $\pi(m|X)$. Moreover, it is easy to see that their multipliers are equal at most in a measure-zero subset of $[0, 1]$.

Therefore, $\pi(h|X), \pi(m|X) \in \{0,1\}$ almost surely. That said, there are two subsets $M, H \in [0,1]$ such that the signals m and h are sent for the members in M and H, respectively. Moreover, define $M^1 = M \cap [0, I)$, $M^2 = [I,1] \cap [I,1]$ and define H^1 and H^2, correspondingly.

If M_1 is empty, then signal m is just sent for a subset of $X \in [I,1]$. In this case, the investor invests even if she receives $(m, \tilde{l})$, which is in contrast with the definition of signal m. Therefore, suppose M_1 is non-empty. For $X \in M_1$ we have $c_m(X) \geq \max\{c_h(X), 0\}$. Rearranging the expressions of $c_h(X)$ and $c_m(X)$, we have

$$\frac{q\lambda^h - (1-q)\lambda^m}{q} \geq \frac{\varepsilon + X - s(X)}{I - s(X)} \geq \lambda^m \Rightarrow q\lambda^h \geq \lambda^m \tag{OA.16}$$

Moreover, note that if M_2 is empty, then the investor does not ever invest when she receives m, which is a contradiction with the definition of m. Therefore, M_2 is non-empty and there exists $X \in M_2$. For X, we have:

$$\begin{aligned} c_m(X) \geq c_h(X) \quad &\Rightarrow \quad (q\lambda^m - (1-q)\lambda^h)(s(X) - I) \geq (1-q)(\varepsilon + X - s(X)) \\ &\Rightarrow \quad q\lambda^m - (1-q)\lambda^h > 0 \end{aligned} \tag{OA.17}$$

Combining (OA.16) and (OA.17), we get $q\lambda^h > \frac{1-q}{q}\lambda^h \Rightarrow q(1+q) > 1$, which contradicts our assumption about the value of q.

□

Lemmas OA5 and OA6 imply that the a two-signal experiment with $\{h, l\}$ must be optimal. In this experiment, the investor completely disregards her own signal. It is easy to see that the optimal two-signal experiment has a threshold scheme, where the threshold $\bar{X}(q)$ should satisfy (OA.11). The rest of the results for this part are similar to Lemma 1.

Proof for Part (b) We can rewrite $\bar{K}_C^\varepsilon(\mu; q) = \mu\mathbb{E}[(s(X) - I)\mathbb{I}_{\{X \geq \max\{\bar{X}(q), \hat{X}(\mu)\}\}}]$, and define

$$\begin{aligned} \mu^l &= \sup\left\{\mu | \bar{K}_S^\varepsilon \text{ is increasing over } [0, \mu]\right\} \\ \mu^h &= \inf\left\{\mu | \bar{K}_S^\varepsilon \text{ is decreasing over } \left[\mu, 1 - \frac{\varepsilon}{I - \bar{X}}\right]\right\} \end{aligned} \tag{OA.18}$$

where $\bar{K}_S^\varepsilon(\mu) = \mathbb{E}\left[(s(X) - I)\mathbb{I}_{\{X \geq \hat{X}(\mu)\}}\right]$. Since $\bar{X}(q) < I - \varepsilon$, there exists $\hat{\mu}(q)$ such that $\bar{X}(q) = \hat{X}(\hat{\mu}(q))$. Because $\bar{X}(q)$ is strictly increasing in q in $[\frac{1}{2}, \bar{q}]$, $\hat{\mu}(q)$ is strictly decreasing in q. If $\hat{\mu}(q) < \mu^l$, then $\bar{K}_C^\varepsilon(\mu; q)$ is increasing over $[0,1]$. If that is not the case, then the function is U-shape over $[\mu^h, 1]$.

Proof for Part (c) Note that as ε goes to zero, $\bar{K}_S^\varepsilon(\mu)$ becomes strictly increasing in $[0,1]$. Therefore, μ^l converges to one as ε goes to zero.

□

In Proposition OA5, the assumption $q \leq \bar{q}$ substantially simplifies the analysis, as the entrepreneur still follows a threshold scheme for disclosure. It is not generally the case, as we show in Lemma B6. In particular, the equilibrium experimentation has a nested-interval structure, consistent with Guo and Shmaya (2019) that finds similar structures under more general settings. For illustration, we take $\mu = 1$ in the next lemma and corollary, the case with interim competition or when $\mu < 1$ are similar.

Lemma OA7. *For $q > \bar{q}$, there exists $q^* > \bar{q}$ such that for $q \geq q^*$, the entrepreneur optimally uses three signals $\{l, m, h\}$. The investor continues the project iff she observes $(m, \tilde{h})$, $(h, \tilde{l})$ or $(h, \tilde{h})$. Specifically, there are $X_m^L(q), X_h^L(q), X_h^H(q) \in (0,1)$ such that the entrepreneur sends h in $[X_h^L(q), X_h^H(q)]$ and m in $[X_m^L(q), X_h^L(q)) \cup (X_h^H(q), 1]$.*

Proof. First, we show that the two-signal experiment introduced in Proposition OA5(a) is not optimal for big enough values of q. Then, we show that when a three-signal experiment is optimal, a nested interval structure is used for providing endogenous information.

Non-optimality of two-signal experiments. Suppose the contrary holds that a threshold scheme, with threshold $\bar{X}(q)$ (as introduced in (OA.11)) is optimal. Then Lemma A1 says it is the optimal experiment among all two-signal experiments that the high signal always induces investment. Now consider η_1, η_2 and η_3 that satisfy the following conditions:

$$q \int_{\bar{X}(q)}^{\bar{X}(q)+\eta_2} (s(X) - I) f(X) dX + (1-q) \int_{1-\eta_3}^{1} (s(X) - I) f(X) dX \leq 0$$

$$(1-q) \int_{\bar{X}(q)-\eta_1}^{\bar{X}(q)+\eta_2} (s(X) - I) f(X) dX + q \int_{1-\eta_3}^{1} (s(X) - I) f(X) dX \geq 0$$

If the two-signal experiment is optimal, then the following three-signal experiment should implement a suboptimal investment function for the entrepreneur.

$$\tilde{\pi}(h|X) = \begin{cases} 1 & X \in [\bar{X}(q) + \eta_2, 1 - \eta_3) \\ 0 & X \in [0, \bar{X}(q) + \eta_2) \cup [I - \eta_3, 1] \end{cases} \qquad \tilde{\pi}(m|X) = \begin{cases} 1 & X \in [\bar{X}(q) - \eta_1, \bar{X}(q) + \eta_2) \cup [1 - \eta_3, 1] \\ 0 & X \in [0, \bar{X}(q) - \eta_1) \cup [\bar{X}(q) + \eta_2, 1 - \eta_3) \end{cases}$$

Therefore, $\eta_1 = \eta_2 = \eta_3 = 0$ should be the solution to the following optimization problem:

$$\max_{\eta_1, \eta_2, \eta_3} (1-q) \int_{\bar{X}(q)-\eta_1}^{\bar{X}(q)} (\varepsilon + X - s(X)) f(X) dX - (1-q) \int_{\bar{X}(q)}^{\bar{X}(q)+\eta_2} (\varepsilon + X - s(X)) f(X) dX - q \int_{1-\eta_3}^{1} (\varepsilon + X - s(X)) f(X) dX$$

$$\begin{aligned} s.t. \quad & q \int_{\bar{X}(q)}^{\bar{X}(q)+\eta_2} (s(X) - I) f(X) dX + (1-q) \int_{1-\eta_3}^{1} (s(X) - I) f(X) dX \leq 0 \\ & (1-q) \int_{\bar{X}(q)-\eta_1}^{\bar{X}(q)+\eta_2} (s(X) - I) f(X) dX + q \int_{1-\eta_3}^{1} (s(X) - I) f(X) dX \geq 0 \end{aligned}$$

Suppose κ_1 and κ_2 are the Lagrange multipliers for the above constraints. The FOCs at $\eta_1 = \eta_2 = \eta_3 = 0$ are as follows:

$$\begin{aligned} [\eta_1]|_{\eta_1=0} = 0 \Rightarrow & f(\bar{X}(q))[(1-q)(\varepsilon + \bar{X}(q) - s(\bar{X}(q)) + \kappa_2(s(\bar{X}(q)) - I))] = 0 \\ \Rightarrow & \kappa_2 = \frac{\varepsilon + \bar{X}(q) - s(\bar{X}(q))}{I - s(\bar{X}(q))} \end{aligned} \tag{OA.19}$$

$$[\eta_2]|_{\eta_2=0} = 0 \Rightarrow f(\bar{X}(q))[-q(\varepsilon + \bar{X}(q) - s(\bar{X}(q))) + q\kappa_1(s(\bar{X}(q)) - I) + (1-q)\kappa_2(s(\bar{X}(q)) - I)] = 0$$
$$\Rightarrow -\kappa_1 = \frac{2q-1}{q}\frac{\varepsilon + \bar{X}(q) - s(\bar{X}(q))}{I - s(\bar{X}(q))} = \frac{2q-1}{q}\kappa_2 \tag{OA.20}$$

$$[\eta_3]|_{\eta_3=0} = 0 \Rightarrow f(1)[-(1-q)(\varepsilon + 1 - s(1)) + (\kappa_1(1-q) + \kappa_2 q)(s(1) - I)] = 0$$
$$\Rightarrow \frac{\varepsilon + 1 - s(1)}{s(1) - I} = \frac{\kappa_1(1-q) + \kappa_2 q}{1-q} = \frac{3q^2 - 3q + 1}{q(1-q)}\frac{\varepsilon + \bar{X}(q) - s(\bar{X}(q))}{I - s(\bar{X}(q))} \tag{OA.21}$$

Note that:

$$\frac{\varepsilon + \bar{X}(q) - s(\bar{X}(q))}{I - s(\bar{X}(q))} \geq \frac{\varepsilon}{I}$$

Therefore, (OA.21) implies that for every $q \in [\frac{1}{2}, 1]$, the following inequality holds:

$$\frac{I}{\varepsilon}\frac{\varepsilon + 1 - s(1)}{s(1) - I} \geq \frac{3q^2 - 3q + 1}{q(1-q)} \tag{OA.22}$$

The RHS in (OA.22) goes to infinity as $q \to 1$, while the LHS is constant. It is the contradiction with the earlier assumption that the two-signal experiment is optimal. Therefore, a three-signal experiment is optimal for large enough values of q.

Nested interval structure. Guo and Shmaya (2019) prove the second part of the lemma in their Theorem 3.1 and Discussion 6.3, for securities $s(X)$ such that $\frac{s(X)-I}{\varepsilon+X-s(X)}$ is increasing in X. Note that this condition holds for special case of $s(X) = X$.

□

Corollary OA1. *For $q \geq q^*$, $\mathcal{I}^q(X) < 1$ for a positive measure of $X \in [I, 1]$, implying a positive probability of inefficient termination. Moreover the investor's interim rent, $\bar{K}_C^{\varepsilon}(q, 1)$, is non-monotone over the region $(\bar{q}, 1]$.*

Corollary OA1 provides a counter-intuitive result that the investor's payoff is not globally increasing in investor sophistication. In fact, if the information the investor receives is very accurate, then the entrepreneur adopts a less informative experiment to increase the chance of inefficient continuations with the cost of a positive probability of inefficient terminations. Similar to Proposition 5 in Kolotilin (2018), Corollary OA1 highlights a "crowding-out effect" of higher levels of the investor's sophistication on the entrepreneur's information production. This effect implies that the investor might be better off by committing to ignoring some part of her independent information, which constitutes an interesting topic for future research.

In the following proposition, we show that while interim competition ($\mu < 1$) can improve the efficiency of investment decisions, it monotonically decreases the insider's interim rent for reasonably sophisticated investors, consistent with Petersen and Rajan (1995).

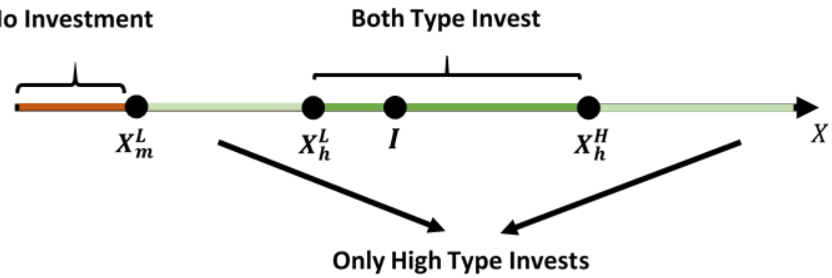

Figure OA2: The nested interval structure of the entrepreneur's experimentation

Proposition OA6 (Impact of Competition (extended))**.**

(a) For a given security with an interim renegotiation, the equilibrium investment function becomes more socially optimal as interim competition increases (μ decreases).

(b) There exists $q^H < 1$ such that for $q \in (q^H, 1)$, the insider's interim rent is monotone in μ. Therefore, the insider never benefits from interim competition for very high levels of sophistication.

Proof. **Proof for Part (a)**

Suppose $\mu_2 > \mu_1$ and the equilibrium investment functions are respectively $\mathcal{I}_2(X)$ and $\mathcal{I}_1(X)$. We show that:

$$\mathbb{E}[(\varepsilon + X - I)\mathcal{I}_1(X)] \geq \mathbb{E}[(\varepsilon + X - I)\mathcal{I}_2(X)] \tag{OA.23}$$

Suppose the contrary. Note that both $\mathcal{I}_1(X)$ and $\mathcal{I}_2(X)$ are implementable for the entrepreneur (there are experiments that implement these investment functions), since q is fixed. The optimality of the investment functions implies:

$$\mathbb{E}[(\varepsilon + (1-\mu_1)(X - I)\mathcal{I}_1(X)] \geq \mathbb{E}[(\varepsilon + (1-\mu_1)(X - I)\mathcal{I}_2(X)] \tag{OA.24}$$

Therefore, if (OA.23) does not hold, (OA.24) implies:

$$\mathbb{E}[(X - I)\mathcal{I}_2(X)] > \mathbb{E}[(X - I)\mathcal{I}_1(X)] \tag{OA.25}$$

Furthermore, the fact that the investor receives positive interim payoff for $q > \frac{1}{2}$ and optimality of $\mathcal{I}_2(X)$ for μ_2 implies:

$$\mathbb{E}[(\varepsilon + (1-\mu_1)(X - I)\mathcal{I}_2(X)] > \mathbb{E}[(\varepsilon + (1-\mu_2)(X - I)\mathcal{I}_2(X)] \geq \mathbb{E}[(\varepsilon + (1-\mu_2)(X - I)\mathcal{I}_1(X)] \tag{OA.26}$$

Therefore, (OA.24) and (OA.26) result in:

$$\begin{aligned} &\mathbb{E}[(\varepsilon + (1-\mu_1)(X - I)\mathcal{I}_1(X)] - \mathbb{E}[(\varepsilon + (1-\mu_2)(X - I)\mathcal{I}_1(X)] \\ \geq &\mathbb{E}[(\varepsilon + (1-\mu_1)(X - I)\mathcal{I}_2(X)] - \mathbb{E}[(\varepsilon + (1-\mu_2)(X - I)\mathcal{I}_2(X)] \\ \Rightarrow &\mathbb{E}[(X - I)\mathcal{I}_1(X)] \geq \mathbb{E}[(X - I)\mathcal{I}_2(X)] \end{aligned} \tag{OA.27}$$

which contradicts (OA.25). Therefore, (OA.23) holds. It shows the equilibrium investment function becomes more socially efficient as μ decreases.

Proof for Part (b)

As it is shown in Lemma OA7, there exists q^* such that for $q > q^*$, a three-signal experiment is optimal. Fix a level of sophistication in the range q. Therefore, there are functions $M_l(\mu)$, $H_l(\mu)$ and $H_h(\mu)$ such that the entrepreneur sends a high signal for $X \in [H_l(\mu), H_h(\mu)]$ and a medium signal in $[M_l(\mu), H_l(\mu)] \cup [H_h(\mu), 1]$. Moreover, interim competition does not affect the entrepreneur's experimentation if $\varepsilon + (1-\mu)(M_l(1) - I) \geq 0$. Therefore, without loss of generality, we assume μ is small enough that $M_l(\mu) = I - \frac{\varepsilon}{1-\mu}$.

Moreover, as discussed earlier, the insider does not get any interim rent conditional on the realization of a medium signal. Therefore, the insider's interim rent is as follows:

$$K_C(\mu; q) = \mu \int_{H_l(\mu)}^{H_h(\mu)} (X - I) f(X) dX = \frac{(2q-1)\mu}{q} \int_{I}^{H_h(\mu)} (X - I) f(X) dX \tag{OA.28}$$

where the last equality comes from the fact that the constraint for sending a high signal is binding. The following lemma characterizes the derivative with respect to μ when $M_l(\mu) = I - \frac{\varepsilon}{1-\mu}$.

Lemma OA8. *If $\varepsilon + (1-\mu)(M_l(1) - I) < 0$, then*

$$\frac{\partial}{\partial \mu} K_C(\mu; q) = \frac{2q-1}{q} \left[\int_{I}^{H_h} (X - I) f(X) dX - \frac{q(1-q)}{2q-1} \frac{\mu \varepsilon^2}{(1-\mu)^3} f(I - \frac{\varepsilon}{1-\mu}) \right] \tag{OA.29}$$

Proof. By taking derivative from (OA.28) with respect to μ, we get:

$$\frac{\partial}{\partial \mu} K_C(\mu; q) = \frac{(2q-1)}{q} \int_{I}^{H_h(\mu)} (X - I) f(X) dX + H_h'(\mu) \frac{(2q-1)\mu}{q} (H_h(\mu) - I) f(H_h(\mu)) \tag{OA.30}$$

We know that the conditions for sending high and medium signal binds for all values of μ. Therefore:

$$\begin{aligned} &\frac{\partial}{\partial \mu} \left\{ (1-q) \int_{I - \frac{\varepsilon}{1-\mu}}^{H_l} (X - I) f(X) dX + q \int_{H_h}^{1} (X - I) f(X) dX \right\} = 0 \\ &\Rightarrow - \frac{(1-q)\varepsilon^2}{(1-\mu)^3} f(I - \frac{\varepsilon}{1-\mu}) + (1-q) H_l'(H_l - I) f(H_l) - q H_h'(H_h - I) f(H_h) = 0 \end{aligned} \tag{OA.31}$$

$$\begin{aligned} &\frac{\partial}{\partial \mu} \left\{ q \int_{H_l}^{I} (X - I) f(X) dX + (1-q) \int_{I}^{H_h} (X - I) f(X) dX \right\} = 0 \\ &\Rightarrow (1-q) H_h'(H_h - I) f(H_h) + q H_l'(H_l - I) f(H_l) = 0 \end{aligned} \tag{OA.32}$$

By combining (OA.31) and (OA.32), we get:

$$H_h'(H_h - I) f(H_h) = - \frac{q(1-q)}{2q-1} \frac{\varepsilon^2}{(1-\mu)^3} f\left(I - \frac{\varepsilon}{1-\mu} \right)$$

We get (OA.29) by substituting the last equality in (OA.30). □

Note that H_h and H_l are the solution to the following maximization problem, where $M_l = I - \frac{\varepsilon}{1-\mu}$:

$$\begin{aligned}\max_{H_l,H_h} \quad & (1-q)\int_{M_l}^{H_l}(\varepsilon+(1-\mu)(X-I))f(X)dX + \int_{H_l}^{H_h}(\varepsilon+(1-\mu)(X-I))f(X)dX \\ & +q\int_{H_h}^{1}(\varepsilon+(1-\mu)(X-I))f(X)dX \\ s.t. \quad & (1-q)\int_{M_l}^{H_l}(X-I)f(X)dX + q\int_{H_h}^{1}(X-I)f(X)dX \geq 0 \\ & q\int_{H_l}^{I}(X-I)f(X)dX + (1-q)\int_{I}^{H_h}(X-I)f(X)dX \geq 0\end{aligned}$$

According to Lemma OA7, we know that both constraints bind. It implies $H_l \to I$ and $H_h \to 1$, as $q \to 1$. Since $f(.)$ and $\frac{\mu}{(1-\mu)^3}$ are bounded ($\mu < 1 - \frac{\varepsilon}{I}$), then the result follows from Lemma OA8.

□

OA6. *Conventional effort distortion*

We have emphasized the entrepreneur's role as an information provider, whereas earlier studies concern the entrepreneur's costly effort to improve project cash flows.[1] We now discuss how these two actions interact.

To model effort, we assume the entrepreneur can choose from a set of conditional distributions $f(X|e)$, where $e \in \mathcal{E}$ is the set of available levels of effort. Function $c(\cdot) : \mathcal{E} \to \mathcal{R}$ shows the cost associated with each level of effort. First, suppose $\mathcal{I}(X;e)$ is the equilibrium investment function when effort level e is chosen. Then, $e^* \in \mathcal{E}$ is *constrained first best* if it solves the following maximization problem:

$$e^* \in \quad argmax \;\; \mathbb{E}[(X - I + \varepsilon)\mathcal{I}(X;e)|e] - c(e) \tag{OA.33}$$

It is straightforward to show that absent investor sophistication and interim competition, there is no effort distortion: given the equilibrium information structure for each level of effort, the entrepreneur chooses the one that is socially preferred. Neither the insider's information monopoly nor IPH distorts entrepreneurial effort. The reason is that when the insider investor gets no rent, the entrepreneur fully internalizes the benefit and the cost of effort. Naturally, in the presence of a sophisticated investor and interim competition, the investor may get positive interim rent that distorts effort provision, but still to a lesser extent compared to the case where information production is exogenous and the insider enjoys full monopoly rent.

OA7. *Scalable investment and continuum range of actions*

Instead of the binary investment decision, we now allow scalable investment at $t = 1$ after observing the experimentation outcome.

Specifically, we assume that investing θI generates a stochastic cash flow $r(\theta)X$, where $r(\cdot) : [0,1] \to [0,1]$ is weakly increasing. For consistency with the baseline model, we assume $r(0) = 0$ and $r(1) = 1$; our baseline

[1] Although extant studies on the agency issues in costly experimentation such as Hörner and Samuelson (2013) and Bergemann and Hege (1998) consider information-acquisition effort, the principal cannot isolate information produced by the agent (hidden effort and hidden information).

model corresponds to $r(\theta) = 0$ for $\theta < 1$ and $r(1) = 1$. Without any interim competition ($\mu = 1$), the final payoffs following investment level θ and realization of cash flow $r(\theta)X$ are $u^E = r(\theta)X - s(\theta, r(\theta)X) + r(\theta)\varepsilon$ and $u^I = s(\theta, r(\theta)X) - \theta I$ respectively, where $s(\cdot, \cdot)$ is a security payment that is generally contingent both on the project scale (or equivalently, level of investment) and the final cash flow, and private benefit of continuation depends on project scale.

For simplicity, we only require the entrepreneur's experimentation to satisfy Bayes Plausibility condition (Kamenica and Gentzkow, 2011) and allow him to choose experiments with infinitely many signals. In particular, we show there are situations that the entrepreneur perfectly discloses X to the investors, since their action set is not finite anymore.

First note that if $\frac{r(\theta)}{\theta}$ is weakly increasing in $\theta \in (0, 1]$, that is, the project has the full benefit of scale, the investment decision reduces to a binary decision and apparently all previous results apply. As such, we focus on the case with diminishing return to scale (DRS). Proposition OA7 extends our main results to this case.

Proposition OA7 (Scalable Investment).

(a) The social welfare of investment, $\mathbb{E}[r(\theta)(\varepsilon + X) - \theta I]$, is increasing in interim competition $1 - \mu$. In particular, when the entrepreneur issues equities, i.e., $s(\theta, r(\theta)X) = \beta r(\theta)X$ for some $\beta \in (0, 1)$ and $r(\theta) = \theta^\gamma$ for some $\gamma \in (0, 1)$, the informativeness of the entrepreneur's experiment weakly decreases in the Blackwell sense, as μ increases.

(b) Suppose $\theta^*(X; \varepsilon)$ is the optimal level of investment for X and ε, i.e. it is the solution of $\max_\theta r(\theta)(X + \varepsilon) - \theta I$. If $r(\theta^*(X; \varepsilon))X - \theta^*(X; \varepsilon)I \geq 0$ with probability one for all values of $\varepsilon \in (0, \bar{\varepsilon})$, then a convertible security for insiders and the residual claim for outsiders implement the first-best outcome, with the entrepreneur choosing the scale after the realization of signal z to achieve the desirable level of investment, and perfectly disclosing information.

Proof. **Proof for Part (a)**

We first characterize the entrepreneur's information design problem and show how the optimal experiment, if exists, changes with μ. For a given security $s(\theta, r(\theta)X)$ and the entrepreneur's experiment $(\mathcal{Z}, \pi)$, let $F_\pi(dz)$ denote the implied measure of unconditional probabilities over signals $z \in \mathcal{Z}$. We further denote the insider's optimal and the socially optimal action given the signal z by $\theta^I(z)$ and $\theta^S(z)$ respectively. In other words,

$$\begin{aligned} \theta^I(z) &\in argmax_{\theta \in [0,1]} \quad \mathbb{E}[s(\theta, r(\theta)X) - \theta I | z], \\ \theta^S(z) &\in argmax_{\theta \in [0,1]} \quad \mathbb{E}[r(\theta)(X + \varepsilon) - \theta I | z] \quad s.t. \quad \mathbb{E}[s(\theta, r(\theta)X) - \theta I | z] \geq 0. \end{aligned} \tag{OA.34}$$

When the signal is privately observed by the insider, she chooses $\theta^I(z)$. When the signal is publicly observed, all investors invest at the welfare-maximizing level, provided the security covers the investment cost. Thus, the entrepreneur solves the following optimization problem

$$\begin{aligned} \max_{(\mathcal{Z}, \pi)} \mu \int_{z \in \mathcal{Z}} & \mathbb{E}[r(\theta^I(z))(X + \varepsilon) - s(\theta^I(z), r(\theta^I(z))X) | z] F_\pi(dz) \\ &+ (1 - \mu) \int_{z \in \mathcal{Z}} \mathbb{E}[r(\theta^S(z))(X + \varepsilon) - \theta I | z] F_\pi(dz) \end{aligned} \tag{OA.35}$$

Equation (OA.35) shows that the entrepreneur faces a trade-off between gaining more rent when the signal is private (the first term) and increasing the efficiency of investment (the second term). As μ increases, the entrepreneur puts smaller weight on the social efficiency of investment, which clearly leads to experiments implementing less socially efficient outcomes.

Now, we characterize the optimal experiment when the entrepreneur uses equities ($s(\theta, r(\theta)X) = \beta r(\theta)X$) and $r(\theta) = \theta^{\gamma}$. First we find the investors' optimal level of investment given a posterior $f(X|z)$. Following the definition of $\theta^I(\cdot)$ and $\theta^S(\cdot)$ provided in (OA.34), we can easily see:

$$\theta^I(z) = \left(\frac{\gamma\beta\mathbb{E}[X|z]}{I}\right)^{\frac{1}{1-\gamma}}$$
$$\theta^S(z) = \min\left\{\left(\frac{\gamma\mathbb{E}[X+\varepsilon|z]}{I}\right)^{\frac{1}{1-\gamma}}, \left(\frac{\beta\mathbb{E}[X|z]}{I}\right)^{\frac{1}{1-\gamma}}\right\}$$

Moreover, since the scale of the investment only depends on the expected state $\mathbb{E}[X|z]$, we can WLOG assume that the signal only has information about the expected state. We can then denote the distribution of signals by $G(X)$, where $F(X)$ is a mean-preserving spread of $G(X)$. As a result, we can rewrite the entrepreneur's problem as follows:

$$\max_{G(X)\prec_{SOSD}F(X)} \int_0^1 u_\mu(X)dG(X), \tag{OA.36}$$

where

$$u_\mu(X) = \mu a_I(X) + (1-\mu)a_S(X)$$
$$a_I(X) = (\frac{\gamma\beta}{I})^{\frac{\gamma}{1-\gamma}} X^{\frac{\gamma}{1-\gamma}}((1-\beta)X+\varepsilon)$$
$$a_S(X) = \begin{cases} \left(\frac{\beta}{I}\right)^{\frac{\gamma}{1-\gamma}} X^{\frac{\gamma}{1-\gamma}}((1-\beta)X+\varepsilon) & \text{if } \beta X \le \gamma(X+\varepsilon) \\ I^{-\frac{\gamma}{1-\gamma}}(\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}})(X+\varepsilon)^{\frac{1}{1-\gamma}} & \text{if } \beta X > \gamma(X+\varepsilon) \end{cases}$$

The following lemma is useful in showing $G(\cdot)$ becomes less informative in the Blackwell sense as μ increases.

Lemma OA9. *For every $\mu \in [0,1]$, there exists $X^*(\mu)$ such that the optimal experiment entails pooling all the types below $X^*(\mu)$ and separating all the types above $X^*(\mu)$.*

Proof. First we show that there exists a threshold value $T(\mu)$ such that $u''_\mu(X) > 0$ for $X > T(\mu)$ and $u''_\mu(X) < 0$ for $X < T(\mu)$. Then, the proof follows from Theorem 1 in Dworczack and Martini (2017).

By taking the second order derivative of $a_I(\cdot)$ and $a_S(\cdot)$, we get:

$$\begin{aligned} a''_I(X) &= \left(\frac{\gamma\beta}{I}\right)^{\frac{\gamma}{1-\gamma}}\left(((1-\beta)\left((2X^2+\frac{\gamma(2\gamma-1)}{(1-\gamma)^2}\right)X+\frac{\gamma(2\gamma-1)}{(1-\gamma)^2}\varepsilon\right)X^{\frac{3\gamma-2}{1-\gamma}} \\ a_S(X) &= \begin{cases} (\frac{\beta}{I})^{\frac{\gamma}{1-\gamma}}\left(((1-\beta)\left((2X^2+\frac{\gamma(2\gamma-1)}{(1-\gamma)^2}\right)X+\frac{\gamma(2\gamma-1)}{(1-\gamma)^2}\varepsilon\right)X^{\frac{3\gamma-2}{1-\gamma}} & \text{if } \beta X \le \gamma(X+\varepsilon) \\ I^{-\frac{\gamma}{1-\gamma}}\left((\frac{\gamma}{(1-\gamma)^2}\right)\left(\gamma^{\frac{\gamma}{1-\gamma}}-\gamma^{\frac{1}{1-\gamma}}\right)(X+\varepsilon)^{\frac{2\gamma-1}{1-\gamma}} & \text{if } \beta X > \gamma(X+\varepsilon) \end{cases} \end{aligned} \tag{OA.37}$$

For $\gamma \ge \frac{1}{2}$, $u_\mu(\cdot)$ is convex and full-disclosure is optimal. Moreover, if $\gamma(1+\varepsilon) \ge \beta$, $u_\mu(X)$ just scales with the change of μ, hence the optimal information disclosure is independent of μ. For $\gamma < \min\{\frac{1}{2}, \frac{\beta}{1+\varepsilon}\}$, there is $T(\mu)$ such that $\left(2X^2 + \frac{\gamma(2\gamma-1)}{(1-\gamma)^2}\right)X + \frac{\gamma(2\gamma-1)}{(1-\gamma)^2}\varepsilon < 0$ if and only if $X < T'$ over the range of $X \in [0,1]$. Therefore, there is a threshold $T(\mu) \in [\frac{\gamma\varepsilon}{\beta-\gamma}, T']$, for which $u''_\mu(X) < 0$ if and only if $X < T(\mu)$.

□

This characterization of optimal disclosure implies that we only need to show $X^*(\mu)$ is weakly increasing in μ. For $\gamma \geq \frac{1}{2}$, $X^*(\mu) = 0$, hence the statement is straightforward. For $\gamma < \frac{1}{2}$, $X^*(\mu) \in [T(\mu), 1]$, since $u''_\mu(X) < 0$ for $X < T(\mu)$ and they should be pooled. Moreover, note that $X^*(\mu)$ is the solution to $\mathbb{E}[u_\mu(X)|X \leq X^*(\mu)] = u_\mu(\mathbb{E}[X|X \leq X^*(\mu)])$. If such a solution does not exist, then $X^*(\mu) = 1$, i.e. no-disclosure is optimal. Finally, it is easy to check that $\frac{\partial^2}{\partial\mu\partial X}u_\mu(X) < 0$ for $X \in [\frac{\gamma\varepsilon}{\beta-\gamma}, T(\mu)]$. Therefore, $\mathbb{E}[u_{\mu_2}(X)|X \leq X^*(\mu_1)] \leq u_{\mu_2}(\mathbb{E}[X|X \leq X^*(\mu_1)])$ for any $\mu_2 > \mu_1$. Consequently, $X^*(\mu_2) \geq X^*(\mu_1)$.

Proof for Part (b)

Consider $s^I(\theta, r(\theta)X) = \lambda \max\{r(\theta)X, \theta I\}$, where λ is defined such that $\mathbb{E}[r(\theta)X - \theta I] = \frac{K}{\lambda}$. Moreover, suppose the outsiders receive the residue $s^O(\theta, r(\theta)X) = r(\theta)X - s^I(\theta, r(\theta)X)$. We want to show that if the entrepreneur fully discloses X and chooses the optimal level of investment $\theta^*(X;\varepsilon)$, then both the insider and the outsiders are willing to invest their share of investment, which are $\lambda\theta^* I$ and $(1-\lambda)\theta^* I$, respectively.

The insider is always willing to invest θ^* as $s^I(\theta, r(\theta)X) - \lambda\theta I \geq 0$ for all values of θ. Moreover, the assumption in the proposition implies that $r(\theta^*)X \geq \theta^* I$ with probability one. Therefore, the outsiders are also willing to participate because

$$s^O(\theta^*, r(\theta^*)X) - (1-\lambda)\theta^* I = r(\theta^*)X - \lambda \max\{r(\theta^*)X, \theta^* I\} - (1-\lambda)\theta^* I = (1-\lambda)(r(\theta^*)X - \theta^* I) \geq 0$$

It is easy to see that the solution works for all ε and the insider's private experimentation.

□

Proposition OA7(a) extends the main result in Section 3 that the insider's interim rent is not increasing in her information monopoly, to continuous (non-binary) investment decisions. More information monopoly (higher μ) means less alignment of the entrepreneur's payoff with the social planner's, which leads to inefficient information production. However, the scalable investment helps the insider to recover some interim rent, even if she fully holds up the entrepreneur ($\mu = 1$). When the insider chooses the scale of investment from a continuous set, she should get zero expected rent from the marginal level of investment. The DRS assumption implies she gets positive interim rent overall, if the investment takes place.

Proposition OA7(b) extends Proposition 2. To implement the first-best outcome under DRS, the security should not only encourage the entrepreneur to experiment efficiently but also incentivize the insider to scale the project efficiently. We thus choose the insider's security to perfectly align the insider's and the social planner's utilities over different scales of investment.

The key takeaway is that with DRS and a continuum of investment levels, investment decisions are no longer binary (investing I or terminating), and the insider derives partial rent from her informational monopoly. Endogenous information production still leads to reduced rent which potentially renders relationship financing infeasible, consistent with Lemma 1. Moreover, similar to the case of binary investments levels, a contractual solution entailing giving convertible securities to initial investors still achieves the first-best outcome, and is robust to investor sophistication.

OA8. *Security design right*

Section 4 mainly focuses on the entrepreneur's optimal security design problem. It turns out that the insider investor would design the contract and security rather differently.

Proposition OA8 (**Security Design Right**). *Under both investor sophistication and unsophistication, it is more socially efficient that the entrepreneur designs the security.*

Proof. According to Propositions 2 and 5, the entrepreneur's design implements the first-best outcome. It is easy to show that the insider receives $\mathbb{E}[(X-I)\mathbb{I}_{\{X\geq I\}}]$ in her optimal design, which implements investment only for $X \geq I$. Therefore, the insider's optimal design does not incorporate the entrepreneur's private benefit of investment, resulting in a lower social welfare. □

The intuition for Proposition OA8 is the following: In order to raise the initial K, the entrepreneur understands he has to provide at least a minimum amount of expected cash flow to the insider investor. Therefore, among all designs that generate this amount, he chooses the one that makes him commit to the most informative disclosure policy, which maximizes the total surplus. In other words, ex ante he is the residual claimant and his incentives are more aligned with a social planner. Moreover, if the insider has the security design right, she would choose a less socially optimal security, because she does not consider the entrepreneur's private benefit from the investment.

References

Bergemann, Dirk, and Ulrich Hege, 1998, Venture capital financing, moral hazard, and learning, Journal of Banking & Finance 22, 703–735.

Dworczack, Piotr, and Giorgio Martini, 2017, The simple economics of optimal persuasion, Journal of Political Economy Forthcoming.

Farrell, Joseph, and Nancy T Gallini, 1988, Second-sourcing as a commitment: Monopoly incentives to attract competition, The Quarterly Journal of Economics 103, 673–694.

Guo, Yingni, and Eran Shmaya, 2019, The interval structure of optimal disclosure, Econometrica 87, 653–675.

Holmstrom, Bengt, 1979, Moral hazard and observability, The Bell Journal of Economics pp. 74–91.

Hörner, Johannes, and Larry Samuelson, 2013, Incentives for experimenting agents, The RAND Journal of Economics 44, 632–663.

Innes, Robert D, 1990, Limited liability and incentive contracting with ex-ante action choices, Journal of Economic Theory 52, 45–67.

Kamenica, Emir, and Matthew Gentzkow, 2011, Bayesian persuasion, American Economic Review 101, 2590–2615.

Kolotilin, Anton, 2018, Optimal information disclosure: A linear programming approach, Theoretical Economics 13, 607–635.

Petersen, Mitchell A, and Raghuram G Rajan, 1995, The effect of credit market competition on lending relationships, The Quarterly Journal of Economics, 110, 407–443.

Von Thadden, Ernst-Ludwig, 1995, Long-term contracts, short-term investment and monitoring, The Review of Economic Studies 62, 557–575.